Appropriate Stochastic Price Models for the Finnish Stumpage Market

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Abstract: In searching for an appropriate time series model based on historical data we applied unit root tests and considered 12 different continuous-times stochastic models prices of six saw log and pulp wood products from two important species of Scots pine (*Pinus.sylvesteris*) and Norway spruce (*Picea.abies*), as well as, average softwood log prices for annual long run and shorter monthly time series in the Finnish wood market. For each product we conducted a comparative analysis between models on the basis of Akaike's Information criteria (AIC), the mean square error (MSE) of the models after one period of forecasting, and a likelihood ratio test. Parameter estimation was performed by quasi maximum likelihood estimation and local linearization method. The unit root tests results showed that while in the long run the price of softwood is trend stationary, in short run it shows non-stationary behaviour. Our results also showed that the level of effect of state the variable on volatility has a major role in refining a general model in to simpler models. The model with a general form of diffusion and no drift yields the highest AIC for most products, and the diffusion part of the model plays an important role in ranking by AIC, while in ranking by MSE for one period of forecasting, the drift part of models plays important role.

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1. Introduction

The behaviour of price processes is a dominant question in uncertainty literature that deals with a landowner's harvesting decisions in dynamic markets and also in connection with roundwood market efficiency. The two main categories of price processes are stationary and non-stationary price processes. These two types of processes have been acknowledged in earlier harvest decision studies. Insley (2002) used dynamic programming to solve a Wickselian optimal rotation problem. She showed that the assumption of mean-reverting or random walk process has a significant effect on the valuation of a stand and optimal rotation. When the stand is young, critical prices for harvesting the stand under mean-reverting price processes are lower than under geometric Brownian motion (GBM) price processes. Tahvonen and Kallio (2006) applied forest-level and risk-aversion under a stochastic programming approach, they showed that under mean-reverting timber prices the harvest age depends more sensitively on the price, and also the forest has higher value, than under the random walk timber price process. Yoshimoto (2009) showed that the critical prices for harvest, wait or abandon decisions of a stand depend on the behaviour of the price processes. Yoshimoto and Shoji (2002) applied 13 different stochastic models on their monthly data sets of log prices in Japanese markets and showed that each specific log could exhibit a different price behaviour and different density distribution.

Market efficiency and price forecasting studies are other areas in which the behaviour of price processes matter. A non-stationary price process is considered to be as a weak form of an information efficient market (Hultkranz 1995). Unit root tests are the main tools for defining market efficiency. Washburn and Binkley (1990) used a unit root test for the timber market of the south-eastern United States and used monthly, quarterly and annual data. They found that the timber market was efficient (non-stationary). Haight and Holmes (1991) showed that while the monthly price had an autoregressive pattern, the quarterly average of the same data followed the random walk process, and concluded that the average price for longer intervals was biased toward a larger autoregressive coefficient. So the assumption of time series behaviour for the same time-span depends on the time interval of the data. Hultkrantz (1993) found that the pooled quarterly data are stationary and the market of timber stumpage price in the south-eastern United States is inefficient when he applied the Dickey-Fuller test to Washburn and Binkley's (1990) quarterly and annual data. Hultkrantz (1995) studied the long-run annual data of timber rent between 1909 and 1990 in the Swedish wood market and found stationary behaviour, which indicated market inefficiency. Prestemon *et al.* (2004), found that quarterly prices from 1977 to 2002 for 22 submarkets in the southern United States are mostly unit root. According to their results, temporal aggregation has an effect on unit root tests and therefore on conclusions that the researcher makes on the behaviour of prices. Lohmander (1988) studied Finnish timber prices using semi-annual data from 1960 to1982. He applied the autocorrelation function (ACF) and partial autocorrelation function (PACF) of real data and the residual of the applied models and found that price processes are stationary. Later, Hultkrantz (1995) applied Lohmander's data in a Dickey-Fuller (DF) test and reached the same conclusion. Toppinen (1998) used the monthly data of saw logs in the Finnish market from 1985 to 1997 and applied augmented Dickey-Fuller (ADF) test and found the price processes to be non-stationary. Linden and Uusivuori (2000) studied the annual data of the roundwood stumpage price in Finland from 1900 to 1995. According to them, the nominal stumpage price has a non-stationary process but deflating the nominal data by the cost of living index makes the series trend stationary. Leskinen and

Kangas (2001) used the annual data of average timber prices from 1950 to 1996 as well as expert judgment, and due to their low number of observations modelled data as AR (1) process. Hänninen *et al.* (2007) used quarterly data from Finland and three other European countries to study the transmission of price changes. They found the Finnish data from 1995 to 2005 to be non-stationary.

In this study we show that the behaviour of timber prices in the long run[*1] may differ from their behaviour in the short run. Since forest management is a long-term project and the price of timber fluctuates over time it is important to study timber price behaviour in the long and short run and see if a difference exists between price behaviour. Thus our objective in this study is to examine the differences in price processes in the long and short run. In this study, which focuses on the dynamic of roundwood prices in the Finnish market, we first compared two sets of time series for the behaviour of roundwood price in the long and short run. Next we proposed 12 continuous stochastic models for two time series and six wood products in the Finnish market and applied AIC for goodness of fit and mean square error (MSE) for the accuracy of forecasting of each stochastic model and product. The remainder of the paper is organized as follows. The material for the research is described in the second section. In the third section the unit root tests and the stochastic model estimation and rankings are described and in the fourth section we present the discussion and conclusions.

2. Data and methods

We used two time series data sets: annual and monthly. The annual

[*1] Long run and short run are hypothetical measures here and are used only to show the differences between our annual data of 107 years and monthly data of 23 years.

data set is the average yearly price of softwood logs (weighted average of Pine and Spruce saw logs) from 1900 to 2007 which has been deflated by the wholesale index (year 2007 as base year). The monthly data is for Scots pine (*Pinus sylvesteris*) and Norway spruce (*Picea abies*) saw logs and pulpwood from January 1986 to September 2008. The data has been deflated using the whole sale index for domestic goods. Since there is no record for separate timber products before 1986 wecompute weighted average monthly data for softwood logs in order to compare the same product over different time length.

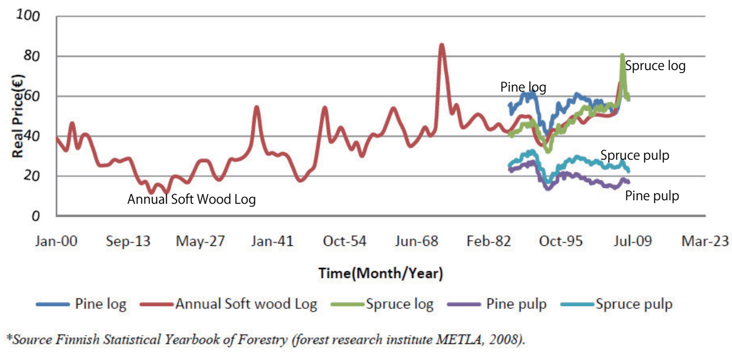

Figure 1. Time series of annual (1900-2007) and monthly (Jan. 1986 – Sep. 2008) roundwood prices in Finland

The concept of stationary time series represents a critical assumption in the analysis of time series data. Its importance lies in the fact that conditions of constant mean variance and covariance are essential in accurately estimating the parameters and models that describe the data. Thus it is important to analyze the data for stationary before using them for modelling and parameter estimation (Metes, 2005). We applied three of the most common tests of unit root. We used an

Augmented Dickey-Fuller (ADF) test which assumes an i.i.d error and null hypothesis of unit root and a Phillops-Perron (PP) test which also has a null hypothesis of unit root but is nonparametric and allows for serial correlation in the innovations. Finally we used the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) that has the null hypothesis trend stationary, as a complementary test (Hamilton 1994.chapter 5).

Table 1. Unit root test for annual and monthly price series of Pine and Spruce pulpwood

Product	Test[1]		t-statistics	Critical t*(5 %)	P value	Conclusions
Annual Softwood	ADF	C&T	-4.315511	- 3.440681	0.0039	Reject H_0
	PP	C&T	- 4.254108	- 3.440681	0.0048	Reject H_0
	KPSS	C&T	0.124657	0.146000	------	Do not reject H_0
Monthly Pine Log	ADF	C&T	- 1.736137	- 3.428123	0.7323	Do not reject H_0
	PP	C&T	- 1.848284	- 3.428049	0.6781	Do not reject H_0
	KPSS	C & T	0.153750	0.146000	------	Reject H_0
Monthly Spruce Log	ADF	C&T	- 1.711098	- 3.428123	0.7437	Do not reject H_0
	PP	C&T	- 1.697166	- 3.428049	0.7499	Do not reject H_0
	KPSS	C&T	0.254249	0.146000	------	Reject H_0
MLS	ADF	C&T	- 3.063425	- 3.490662	0.1173	Do not Reject H_0
	PP	C&T	- 2.545690	- 3.490662	0.3059	Do not Reject H_0
	KPSS	C&T	0.191224	0.146000	------	Reject H_0

[1] For PP and KPSS, lag length has been chosen based on the kernel Bartlett spectrum estimation method and Andrews Bandwidth Selection. For ADF, lag length was selected using the AIC as lag selection criteria.

The tests were carried out on the level and the result for all products and all tests at first difference were stationary. For monthly data the ADF and PP tests' constant and linear trend parameters were not significantly different from zero, but since the result the ADF and PP tests in all conditions were the same and for the KPSS test the constant and trend are different from zero. For comparability, we chose all tests with constant and trend. For the annual data, the unit root tests (ADF and PP) were rejected and the trend stationary KPSS test could not be rejected, so we have confirmatory results at 5% significance that the price processes are stationary. Monthly data, on the other hand, do not reject the unit root tests (ADF and PP) null hypothesis and reject the null hypothesis of the KPSS test, which means that for shorter time-spans of the monthly data the process is non-stationary.

3. Stochastic models

A typical one-dimensional continuous time model used in finance has the following general form:

[1] $$dx_t = f(x_t)dt + g(x_t)dz_t$$

where the first part of the RHS represents the drift of the model and the second represents its diffusion, and dz_t denotes the increment of standard Wiener process or Brownian motion ($dz = \varepsilon\sqrt{\Delta t}$,$\varepsilon \sim N(0,1)$).

In this study we propose a general model with the linear drift term as a function of the state variable and the diffusion term as a non-linear function of the state variable. By adding restriction to this general model we can produce different models. By letting x_t be stumpage price at time t, the general price process model can be defined as:

[2] $$dx_t = (\alpha + \beta x_t)dt + \sigma x_t^\gamma dz_t$$

In equation [2], the drift has two parameters α and β, which guarantee the existence of the mean-reverting models. In the diffusion part, σx_t^γ is the instantaneous standard deviation of log price changes which is often referred as 'volatility'. Parameter γ shows the level of effect of the state variable on the process variance (here after LESV). The dependence of the instantaneous standard deviation on x_t^γ is known as the 'levels effect' (Yoshimoto and Shoji, 2002).

In order to consider specific models from the general model [2], we established restrictions on 3 estimable parameters. That is on the parameters α and β in the drift part of equation [2] and parameter γ in its diffusion part. Based on the restrictions on drift parameters we obtained two main categories of mean-reverting (containing both drift parameters) and non-stationary models as well as four classes of models, and in each class based on restrictions on the value of γ we obtained three models, so in total we have 12 different stochastic estimable models in 4 classes and 2 categories (see Tab.2).

Table 2. Proposed stochastic models

Models	variables	Stochastic models
Model 1	α, β, γ	$dx_t = (\alpha + \beta x)dt + \sigma x^{\gamma} dz$
Model 2	$\alpha, \beta, 1$	$dx_t = (\alpha + \beta x)dt + \sigma x dz$
Model 3	$\alpha, \beta, 0$	$dx_t = (\alpha + \beta x)dt + \sigma dz$
Model 4	$0, \beta, \gamma$	$dx_t = \beta x dt + \sigma x^{\gamma} dz$
Model 5	$0, \beta, 1$	$dx_t = \beta x dt + \sigma x dz$
Model 6	$0, \beta, 0$	$dx_t = \beta x dt + \sigma dz$
Model7	$\alpha, 0, \gamma$	$dx_t = \alpha dt + \sigma x^{\gamma} dz$
Model 8	$\alpha, 0, 1$	$dx_t = \alpha dt + \sigma x dz$
Model 9	$\alpha, 0, 0$	$dx_t = \alpha dt + \sigma dz$
Model 10	$0, 0, \gamma$	$dx_t = \sigma x^{\gamma} dz$
Model 11	$0, 0, 1$	$dx_t = \sigma x dz$
Model 12	$0, 0, 0$	$dx_t = \sigma dz$

Based on the drift restriction we can categorize the 12 models in to 4 classes: Models 1, 2 and 3 in the first class, which had two drift parameter; models 4, 5 and 6 in the second class, which has a drift parameter dependent on the time increment and current price; models 7, 8 and 9 in the third and models 10, 11 and 12 in the fourth class.

Parameter estimation

The parameters are estimated by the local linearization method introduced by Ozaki (1985) and applied in forest economics by Yoshimoto and Shoji (1998, 2002). The basic idea of the method is that an original nonlinear stochastic differential equation is first converted to a stochastic differential equation having a constant diffusion term, and then the nonlinear drift of the derived stochastic differential equation is locally approximated by a linear function of state[*2]. Since the resultant stochastic differential equation is analytically solvable, the corresponding likelihood function for the parameter estimation can be achieved (Yoshimoto and Shoji, 2002).

Although there are other methods for parameter estimation with a finite sample such as least square method and generalized method of moments (GMM), Shoji and Ozaki (1997) indicate that the local linearization method is superior for finite sample performance i.e., bias, variance, and mean square error of variances of the parameter estimates.

We first estimate the mean equation by least squares and use its parameter estimates and residuals as initial values for the ML estimation. Table 3 shows the parameter estimation results.

P-values for parameters α and β indicate that except for model 5 (which is a GBM) we can omit these parameters for the models; in other words class 4 is enough to represent the data. For spruce pulpwood, the LESV value is less than zero and suggests that volatility changes negatively with the price, or, as the price increases, the volatility will decrease. The other products have a positive LESV value which indicates that an increase in price will increase the volatility. Another

[*2] Further discussion on the parameter estimation method can be found in Yoshimoto and Shoji (2002).

Table 3. Estimated parameters

Parameters	Products	Models 1	2	3	4	5	6	7	8	9	10	11	12
α	PL	0.165 [0.113]	0.188 [0.073]	0.0019 [0.040]				0.0086 [0.542]	0.0066 [0.647]	0.18E-03 [0.883]			
	PP	0.107 [0.262]	0.106 [0.268]	0.0072 [0.385]				– 0.035 [0.815]	– 0.046 [0.761]	-0.0010 [0.435]			
	SL	0.0396 [0.553]	0.0664 [0.298]	0.0096 [0.191]				0.0224 [0.100]	0.0220 [0.174]	0.0016 [0.314]			
	SP	0.119 [.348]	0.0974 [.298]	0.0097 [.297]				-0.0057 [.702]	-0.0064 [.964]	-0.42E- 03			
	ASL	0.013 [0.03]	0.011 [0.030]	0.0025 [0.42]				0.0044 [0.056]	0.0045 [0.037]	0.12E-3 [0.738]			
	MSL	0.085 [.908]	.134 [.846]	.0035 [.630]				.374 [.032]	.381 [.041]	.0033 [.091]			
β	PL	-0.161 [0.135]	-0.185 [0.077]	-0.0019 [0.029]	0.0067 [0.646]	0.0024 [0.000]	-.22E-04 [0.984]						
	PP	-0.145 [.233]	-0.143 [.231]	-0.0099 [.289]	-0.0093 [.624]	0.0038 [.000]	-0.0015 [.294]						
	SL	-0.0158 [0.804]	-0.0394 [0.464]	-0.0067 [0.167]	0.0196 [0.134]	0.0022 [0.000]	0.0011 [0.288]						
	SP	-0.120 [0.324]	-0.0983 [0.316]	-0.0098 [0.275]	-0.0073 [0.611]	0.0025 [0.000]	-0.55E- 03						
	ASL	-0.0064 [0.141]	-0.0518 [0.219]	-0.0014 [0.017]	0.0019 [0.286]	0.0016 [0.000]	-0.008 [0.615]						
	MSL	.244 [.705]	.205 [.717]	-.00085 [.984]	.313 [.039]	.028531 [.000]	.0025 [.032]						
γ	PL	1.47 [0.000]	1	0	1.50 [0.000]	1		1.51 [0.000]	1	0	1.49 [0.000]	1	
	PP	1.15 [0.000]	1	0	1.14 [0.000]	1		1.15 [0.000]	1	0	1.16 [0.000]	1	
	SL	1.59 [0.000]	1	0	1.60 [.000]	1		1.59 [0.000]	1	0	1.59 [0.000]	1	
	SP	-0.188 [0.337]	1	0	-0.189 [0.282]	1		-0.187 [0.287]	1	0	-0.177 [0.317]	1	
	ASL	0.87 [0.000]	1	0	0.923 [0.000]	1		0.928 [0.000]	1	0	0.892 [0.000]	1	
	MSL	1.22 [.000]	1	0	1.22 [.000]	1		1.22 [.000]	1		1.21 [.000]	1	
σ^2	PL	0.00465 6	0.004702 [0.000]	0.005034 [.000]	0.004624 [.000]	0.004664 [.000]	0.005082 [.000]	0.004626 [.000]	0.004666 [.000]	0.005081 [.000]	0.004622 [.000]	0.0046622 [.000]	0.005082 [.000]
	PP	0.00827 0	0.007786 [0.000]	0.005465 [0.000]	0.008205 [0.000]	0.007755 [0.000]	0.005488 [0.000]	0.008244 [0.000]	0.007758 [0.000]	0.005496 [0.000]	0.008281 [0.000]	0.007757 [0.000]	0.005508 [0.000]
	SL	0.00328 5	0.004169 [.000]	0.007744 [.000]	0.003273 [.000]	0.004157 [.000]	0.007776 [.000]	0.003279 [.000]	0.004160 [.000]	0.007767 [.000]	0.003298 [.000]	0.004176 [.000]	0.007797 [.000]
	SP	0.00516 0	0.005098 [.000]	0.005061 [.000]	0.005132 [.000]	0.005074 [.000]	0.005079 [.000]	0.005130 [.000]	0.005075 [.000]	0.005081 [.000]	0.005129 [.000]	0.005073 [.000]	0.005083 [.000]
	ASL	0.02107 1	0.033254 [0.000]	0.001136 [0.000]	.024442 [0.008]	0.031894 [0.000]	0.001204 [0.000]	0.025001 [0.010]	0.032531 [0.000]	0.001205 [0.000]	0.02137 [0.006]	0.031145 [0.000]	0.001206 [0.000]
	MSL	.046542 [0.000]	.052340 [.000]	.115284 [.000]	.046508 [.000]	.052298 [.000]	.115352 [.000]	.046673 [.000]	.052437 [.000]	.115284 [.000]	.047159 [.000]	.052941 [.000]	.116628 [.000]

*p values in the brackets

result is that the higher the absolute LESV value the lower will be the value of volatility which is in line with previous study of Yoshimoto and Shoji's (2002) study.

Comparison of model performance

In order to rank the performance of each of the stochastic models, we applied Akaike's Information Criteria(AIC), which can be derived from the log likelihood function of the models, AIC= -2*(LOGL+NOP), where LOGL is the maximum value of log likelihood and NOP the number of parameters in each stochastic model. It is known that, for a given data set, larger number of model parameters implies larger likelihood value, which yields an incorrect assessment of the results for the model selection. AIC was devised to overcome this problem, so we used it as a comparison criterion.

Table 4. Sorting 12 stochastic differential equations by AIC

Rank	Products					
	Pine Log	Pine Pulp	Spruce Log	Spruce Pulp	ASL	MSL
1	10 (-1856.68788)	10 (-1826.30398)	7 (-1808.94269)	12 (-1831.78360)	8 - 642.39	4 -687.13033
2	7 (-1855.06484)	2 (-1826.07959)	4 (-1808.77789)	10 (-1830.11377)	2 - 642.3	7 -687.03747
3	4 (-1854.90667)	8 (-1826.06006)	10 (-1808.55476)	6 (-1829.99620)	7 - 641.2	8 -686.64220
4	1 (-1854.65728)	4 (-1824.55198)	1 (-1806.98254)	9 (-1829.89386)	1 - 641.1	1 -685.13888
5	8 (-1854.04673)	1 (-1824.38837)	8 (-1797.75074)	3 (-1828.97435)	10 - 640.1	10 -684.84979
6	2 (-1853.99988)	7 (-1824.36159)	2 (-1795.98324)	4 (-1828.37168)	4 - 639.3	2 -684.71553
7	5 (-1851.92572)	5 (-1824.28700)	5 (-1795.47627)	7 (-1828.25981)	5 - 639	5 -684.69480
8	11 (-1843.11446)	12 (-1810.01483)	11 (-1786.66973)	1 (-1827.33902)	3 - 612.5	11 -670.64266
9	12 (-1831.87637)	6 (-1809.03756)	12 (-1715.84257)	8 (-1814.69717)	12 - 610.1	9 -607.86607
10	3 (-1830.40734)	9 (-1808.63849)	9 (-1714.90112)	2 (-1813.62061)	6 - 608.3	6 -607.70746
11	9 (-1829.89808)	3 (-1808.16126)	6 (-1714.58616)	5 (-1812.72894)	9 - 608.2	12 -606.72532
12	6 (-1829.87671)	11 (-1806.21722)	3 (-1713.70070)	11 (-1802.67274)	11 - 606.1	3 -1605.86624

AIC showed that for pine and spruce saw logs the best models were models with unrestricted LESV (models 10, 7, 4, 1) and the worst those constant volatility (models 12, 9, 6 and 3). For pine pulpwood, the best models were those with unit LESV ($\gamma = 1$) and the worst those with constant volatility ($\gamma = 0$). For spruce pulp, on the other hand, the best-fitted models were those with constant volatility and the worst those with linear diffusion (models 8, 2, 5, 11). AIC ranking was also has a relation with the value of parameter LESV. Spruce pulp had a negative LESV and hence the best-fitted model for that product was the one with constant volatility (model 12). For other products with a positive and close to or bigger than 1 LESV value models with constant volatility were among the least favourite.

For long-run annual data the three best models were those with unit LESV (Models 8 and 2). While for the average monthly data of softwood the best models were 4 and 7. For this product the drift also had an important effect on the ranking. For instance, a model with one drift parameter which shows the drift depends on time increment and restricted linear LESV was the best. The second-best model was model 2 with two drift parameters and restricted linear LESV. Thus model 5 which has one drift parameter (like model 8) and restricted unit LESV (like models 2 and 8) moved to the 7th place in the ranking table because its drift is price dependent. In general value of LESV is an important factor in the AIC ranking. Table 4 also shows that some models for each product had a very close AIC value, so we also investigated whether a significant difference existed between the general model and the other models. Since the restrictions hierarchically imposed on the general model the likelihood ratio test applied to determine if there was a significance difference between the general model and the other stochastic models.

The likelihood ratio test results are presented in Table 5. For pine products the last five models in the ranking of table 3 (models 11, 12, 3, 9, 6) could not be accepted as not being significantly different from general model of equation [1]. For spruce logs only 3 models at the top of ranking were accepted as not being significantly different from general model. For spruce pulpwood, models with linear relation of volatility and current price cannot be accepted (models 2, 5, 8 and 11). For average annual and monthly data models with constant volatility cannot be accepted.

Table 5. Likelihood Ratio Test results for general model compared to the other 11 models

Models	Degree of Freedom	Products Pine Log	Pine Pulp	Spruce Log	Spruce pulp	ASL	MSL
2	1	0.1030	0.5777	0.0030*	0.0000*	0.3833	.1156
3	1	0.0000*	0.0002*	0.0000*	0.5452	0.0000*	0.0000*
4	1	0.1812	0.1752	0.6515	0.3252	0.0506	0.8034
5	2	0.0939	0.3496	0.0012*	0.0002*	0.1313	0.2868
6	2	0.0000*	0.0006*	0.0000*	0.5170	0.0000*	0.0000*
7	1	0.2068	0.1544	0.8415	0.2987	0.1681	0.6947
8	2	0.0997	0.3119	0.0013*	0.0002*	0.3396	0.2794
9	2	0.0000*	0.0005*	0.0000*	0.4853	0.0000*	0.0000*
10	2	0.3734	0.5051	0.2970	0.5417	0.0837	0.1141
11	3	0.0005*	0.0002*	0.0000*	0.0000*	0.0000*	0.0001*
12	3	0.0002*	0.0010*	0.0000*	0.6694	0.0000*	0.0000*

* *P-values* which reject the null hypothesis of no difference with general model at 5% significance level.

Null: restricted model Alternative: Unrestricted model

* cannot accept the null hypothesis of restriction at a 5% significance level.

The estimated models are also valuable for forecasting the future and we can use forecasting accuracy as a complement to model selecting. We estimated the mean square error (MSE) of each model for one period ahead.

For all products, the first class had the lowest mean square error and for 3 out of 4 products the last class had the highest mean square error. This shows that unlike using AIC criteria where ranking depends mostly on the diffusion part of the model, the MSE criteria ranks the models based more on the drift part of the model. Models with more parameters in the drift part ranked higher and with less or no drift ranked lower.

Table 6. Ranking stochastic models based on Mean Square Error performance

Rank	Products					
	Pine Log	Pine Pulp	Spruce Log	Spruce Pulp	MSL	ASL
1	1 (0.41908)*	1 (0.45517)	1 (0.64330)	1 (0.41962)	3 0.96070	3 (1.136)
2	3 (0.41952)	3 (0.45541)	2 (0.64581)	3 (0.42174)	9 0.96070	1 (1.155)
3	2 (0.41967)	2 (0.45563)	3 (0.64532)	2 (0.42181)	8 0.96091	2 (1.158)
4	7 (0.42321)	4 (0.45737)	6 (0.64798)	4 (0.42323)	2 0.96099	8 (1.189)
5	8 (0.42337)	5 (0.45739)	4 (0.64771)	6 (0.42326)	7 0.96101	7 (1.193)
6	10 (0.42339)	6 (0.45730)	5 (0.64805)	5 (0.42329)	1 0.96102	6 (1.204)
7	4 (0.42339)	7 (0.45790)	7 (0.64677)	8 (0.42337)	4 0.96119	9 (1.205)
8	9 (0.42342)	8 (0.45795)	8 (0.64713)	7 (0.42340)	6 0.96126	12 (1.206)
9	12 (0.42345)	9 (0.45797)	9 (0.64723)	9 (0.42342)	5 0.96127	10 (1.208)
10	6 (0.42345)	10 (0.45823)	10 (0.65047)	12 (0.42360)	12 0.97190	5 (1.210)
11	5 (0.42349)	12 (0.45903)	12 (0.64976)	10 (0.42361)	10 0.97438	4 (1.212)
12	11 (0.96329)	11 (1.4126)	11 (1.3379)	11 (1.08149)	11 2.4304	11 (1.855)

*Values have been multiplied by 1000

Even with small differences in AIC, different stochastic models would lead to a different distribution of future price dynamics. Figure 2 a and b depicts the histogram of the simulated sample path after 20 years of simulation in terms of density of pine saw logs. The number of simulations was 500, the time increment was set to 1/12 y (meaning one month) and the initial index of price level was set to 1. Figure 2a, shows the distribution of best (model 10) and worst (model 6) of AIC performance for pine saw logs while Figure 2b shows the performance of the GBM and GMR models.

4. Conclusions

The stationary test results show that in the long run the timber price

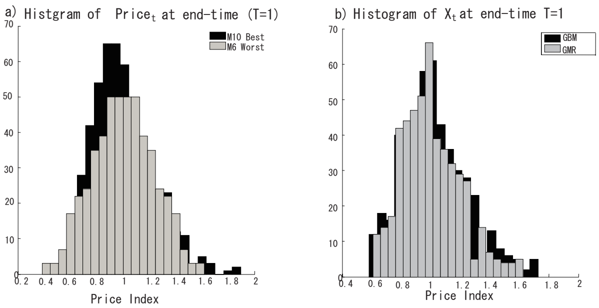

Figure 2. Distribution of simulated best and worst AIC for Pine saw log

Table 7. Descriptive statistics for simulated sample path for Pine saw log

Model	Minimum	Maximum	Mean	Median	SD	Skewness	Kurtosis
10	0.5544	2.012	0.9986548	0.965956	0.218186	1.0045	4.725967
6	0.3254	1.674	0.999	0.9998	0.233100	-9.256e-15	3.037591
GBM	0.5202	1.895	1.02289	1.002522	0.219815	.6386224	3.644128
GMR	0.5462	1.786	1.002409	0.9831988	0.195934	.638622	3.653759

process is stationary, and in the short run non-stationary. This finding is similar to that of Hultkrantz (1995) for long series annual data as well as studies on short series monthly data such as Prestemon (2004) and Toppinen (1998). However, Linden and Uusivuori (2000) arrived at different conclusions for long run time series of nominal timber prices in Finland. The unit root test results confirm that in the long run the effects of sudden shocks disappears while in short run shocks can change the direction of stumpage price processes (Yoshimoto and Kato 2004). This is also in agreement with the economics theory of changes in commodity prices; for instance, as the price of a commodity rises, the supply of that commodity will increase due to the entry of higher cost producers to the market, which will put downward pressure on the price and vice versa. So the price will behave more like a mean reverting process (Dixit and Pindyck, 1994, p69).

For forest stand management several one-dimensional continuous stochastic models have been proposed to capture the uncertainty of prices in long run as well as short run. Since the price processes dynamics will affect forest manager's decisions misspecification of the price model will result in a non-optimal decision under uncertainty (e.g., Yoshimoto and Shoji, 1998, Tahvonen and Kallio, 2006). In the present study the objective was to propose different stochastic models for stumpage price dynamics and conduct a comparative analysis of their performance for long run and short run stumpage price data.

In previous studies on the effect of stochastic price processes on forest management, GBM and mean- reverting processes were often defined for price dynamics (e.g. Haight and Holmes, 1991, Thomson, 1992, Yoshimoto and Shoji, 1998, Insley, 2002, Insley and Rollins, 2005, Yoshimoto, 2009). This is mostly because of the tractability of such stochastic processes and being common in Financial analysis. Although tractability and analytical solution are important, restricting the research on dynamic stochastic analysis to tractable models will raise the question of the efficiency of such stochastic models in capturing price dynamics.

Our results show that the dynamics of price processes for different roundwood products in Finland can be captured by different models and that a GBM or certain model of mean-reverting (e.g. the Ornstein-Uhlenbeck model) may not be the best fit for every price process. In fact, it is better to consider a more general form of stochastic model and refine the model by specifying parameter values.

Our results also show that for most products and monthly data the best-fitted models are model with no drift. This is in contrast with the Japanese market result by Yoshimoto and Shoji (2002), in which for most of their products general form performed better. In addition, comparing long run and monthly data in the case of best-fitted models show that a mean-reverting model fits long run better than short run data.

Further, a relation between AIC ranking and the diffusion part of the stochastic model and LESV value is apparent. Refinement the models can be implemented regarding to the LESV value. LESV shows weight of state variable in diffusion of price processes dynamics, considering processes with constant or linear volatility easier to handle and more practical, one can refine the LESV of less (more) than 0.5 to zero. LESV less than 0.5 shows the low dependence between price and its volatility so it can be refine to zero while, LESV of more than 0.5 shows higher dependency of price to the volatility and can be refine to 1. On the other hand, the MSE for the forecasting of one period ahead has been seen to be influenced more by a functional form of the drift term, where as in our cases more drift term usually produced a better MSE fit.

Our simulation analysis showed that different stochastic models will lead to a different distribution of simulated sample paths, which can consequently affect forest management decisions (e.g. Tahvonen and Kallio, 2006, Insley, 2002). The probability of observing a price which is above or below some given threshold price is dependent on the distribution of future price paths, thus value of a forest management action is dependent on the chosen price processes. Further studies can be directed toward stochastic volatility models and comparison between them. As well investigating the effects of different stochastic volatility models on optimal decision making is necessary.

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