

Carbon Dynamics and Optimal Forest Rotation

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Keywords: Dynamic programming, optimal forest rotation, dead organic matter, boreal forest

Abstract: Carbon sequestration in forests is being considered as a mechanism to slow or reverse the trend of increasing concentrations of carbon dioxide in the atmosphere. This paper presents the results from a dynamic programming model used to determine the optimal harvest decision for a forest stand that provides both timber harvest volume and carbon sequestration services in the Canadian boreal forest. The primary contribution of this study is that the model incorporates detailed information on carbon stocks in dead organic matter pool (DOM), which have been ignored in stand level economic models. It also considers varying levels of initial DOM stocks. The varying DOM stock has interesting implications because it allows one to establish an initial DOM stock for which a landowner will voluntarily participate in the type of carbon market considered in this study. The results of the study indicate that while optimal harvest age seems to be relatively insensitive to carbon stocks in DOM pool, initial carbon stock levels significantly affect economic returns to carbon management.

Received July 31, 2010; Accepted November 30, 2010

1. Introduction

In response to global concern about climate change, policy makers and scientists are searching for ways to slow or reverse the trend of increasing concentrations of greenhouse gases, especially carbon dioxide (CO₂), in the atmosphere. Forests are viewed as potential carbon sinks. As trees grow, photosynthesis converts CO₂ into cellulose and other plant material, temporarily removing it from the atmosphere. In addition, a substantial amount of carbon is stored in forests as dead organic matter (DOM) in standing snags, on the forest floor, and in the soil until the process of decomposition releases it back to the atmosphere.

The Intergovernmental Panel on Climate Change (IPCC) provides guidelines for the calculation and reporting of changes in stocks of forest carbon (IPCC, 2006) as it relates to national greenhouse gas inventories. The IPCC identifies three tiers for reporting changes in stocks of forest carbon. These tiers reflect the relative importance of forest carbon stocks to greenhouse gas inventories and the sophistication of the data collection and monitoring infrastructure of countries. Canada has elected to use tier 3 methodologies (with the most detailed reporting requirements) for reporting changes to carbon stocks on managed forest lands. The IPCC specifies five carbon pools that must be accounted for: above-ground biomass, below-ground biomass, dead wood, litter, and soil carbon. The Canadian Forest Service developed the Carbon Budget Model of the Canadian Forest Sector (CBM-CFS3) to track and report changes in forest carbon stocks (Kull *et al.*, 2007). CBM-CFS3 is a detailed model that recognizes more than 20 different carbon pools within a forest stand and tracks the transfer of carbon between these pools and the atmosphere (Fig.1).

Carbon sequestration services have value and markets are developing to allow for trade in these services. For example, the Chicago Cli-

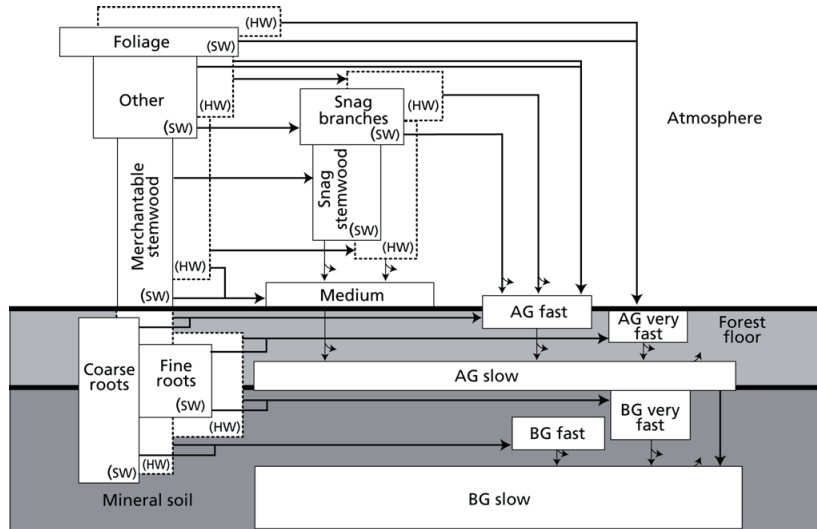


Figure 1. The carbon pool structure of the CBM-CFS3

Very fast, fast, medium, and slow refer to relative decomposition rates for pools. Curved arrows represent transfers of carbon to the atmosphere, and straight arrows represent transfers from one pools to another. SW is softwood, HW is hardwood, AG is above ground, and BG is below ground. Illustration courtesy of the Canadian Forest Service, reproduced with permission from (Kull *et al.*, 2007, Fig.1-1).

mate Exchange (CCX) has developed contracts for voluntary carbon offsets generated through forest management activities (CCX, 2009). However, a component that has not been investigated in detail in the literature on forest carbon sequestration is the role of DOM. As the relationships between DOM, carbon stocks and forest harvesting are complex, detailed investigation of these issues is required to be assured that markets for forest carbon are accurately capturing sequestration dynamics. The purpose of this study is to explore how markets for forest carbon sequestration services could affect optimal forest management decisions when DOM is taken into account.

The classic problem in forest economics is the determination of the harvest age for an even-aged forest stand which maximizes the net present value of an infinite series of timber regeneration, growth, and harvest cycles. Faustmann (1849) is usually attributed with the first correct solution to this problem when only timber values are considered. Samuelson (1976) provides a more formal mathematical specification of the problem. Hartman (1976) extends the model to include values associated with standing trees (e.g. wildlife habitat) as well as the extractive value of timber harvest.

The Hartman model is used by van Kooten *et al.* (1995) in an early exploration of the effect of carbon prices on optimal forest harvest age in western Canada. In their analysis, the amount of carbon stored in the forest stand is proportional to volume of merchantable timber on the site at a particular stand age. The forest owner is paid for the accumulation of carbon in biomass associated with growth, and pays for carbon released to the atmosphere at harvest. Some of the harvested timber is assumed to be permanently stored in structures and landfills. There is no recognition of DOM or soil carbon in the van Kooten analysis.

However, DOM and soil carbon can represent a substantial proportion of the total carbon stored on forested site. Also, management decisions such a harvest age can have a substantial effect on soil carbon stocks (Aber *et al.*, 1978, Kaipainen *et al.*, 2004). Covington (1981) found that forest floor mass declines sharply following harvest, with about 50% of forest floor organic matter lost in the first 20 years. DOM may increase immediately following harvest as a result of slash and other debris left on site (Black and Harden, 1995).

Gutrich and Howarth (2007) develop a simulation model of the economics of timber and carbon management for five different forest types in New Hampshire, USA. Their model includes representation of car-

bon stored in live biomass, dead and downed wood, soil carbon, and wood products (hardwood pulp, hardwood saw, softwood pulp, and softwood saw products). Annual transfers of carbon between pools are modelled using straightforward functions. For each timber type, an initial stock of carbon in the dead and downed wood pool is assumed, and the NPV maximizing model is solved using a grid search procedure given an initial stock of carbon in the non-biomass pools.

At the forest level, McCarney (2007) presents one of the few studies which include a representation of carbon stocks in a DOM pool in a model optimizing the joint value of timber and carbon. He utilized the Carbon Budget Model of the Canadian Forest Sector (CBM-CFS3) to decompose forest carbon stocks into separate biomass and DOM carbon pools. In McCarney's model, DOM is assumed to have a fixed initial DOM stock as opposed to varying initial DOM stocks used in this model.

The contribution of the present study is an extension the work of van Kooten *et al.* (1995) incorporating DOM or soil carbon pool. While Gutrich and Howarth (2007) did include DOM and soil carbon pools in their model, they assumed a fixed initial stock of DOM. This paper extends the Gutrich and Howarth (2007) approach by solving simultaneously for a range of initial DOM stocks. As it is demonstrated later in this paper, the size of the initial DOM stock controlled by a forest owner significantly affects the incentives associated with carbon management and the attractiveness of carbon markets to forest landowners.

In this paper, a dynamic programming model is developed to determine the optimal harvest decision for a forest stand described by two state variables: stand age and carbon stocks in DOM. This model is used to:

1. examine the sensitivity of optimal harvest age to stocks of carbon

- in DOM, carbon prices, and lumber prices,
2. examine the sensitivity of the net present value of forested land to stand age, stocks of carbon in DOM, and carbon prices,
 3. examine projected trajectories of carbon stocks in DOM given optimal harvest rules for different carbon prices, and
 4. examine the impact of ignoring carbon stocks in DOM on the optimal harvest decision.

2. The Model

The basic assumption of the model is that the decision maker (a forest landowner) is participating in a carbon market where the landowner is paid for carbon sequestered by the forest and pays when carbon is released from the forest. The landowner is assumed to manage the forest jointly for timber production and carbon sequestration in a manner that earns maximum discounted financial return. The forest is managed using an even-aged silvicultural system. Each rotation begins with the establishment of a stand on bare forest land and ends with a clearcut harvest after a number of years of growth. The beginning of a new rotation coincides with the end of the previous rotation. The cycle of establishment, growth, and harvest is assumed to repeat *ad infinitum*.

The growth and yield data and the cost information used in this study come from the TIPSYS growth and yield simulator (BC MoFR, 2007) developed by the British Columbia Ministry of Forests. The data represents a lodgepole pine stand in the BWBS biogeoclimatic zone, in the Dawson Creek Forest District of the Prince George Forest Region of British Columbia, Canada. A medium site class (site index = 16 m at 50 years breast height age) and a planting density of 1,600 stems/ha is assumed.

A tabular representation of the merchantable timber yield table from

TIPSY is approximated using a Chapman-Richards growth function:

$$[1] \quad V(t) = v_1(1 - e^{-v_2 t})^{v_3}$$

in which $V(t)$ represents the merchantable timber volume in m^3/ha at age t and v_1 , v_2 and v_3 are parameters, which were set at 500, 0.027 and 4.0 respectively. The Chapman-Richards approximation is a good representation of the yield table generated by TIPSY (Fig.2).

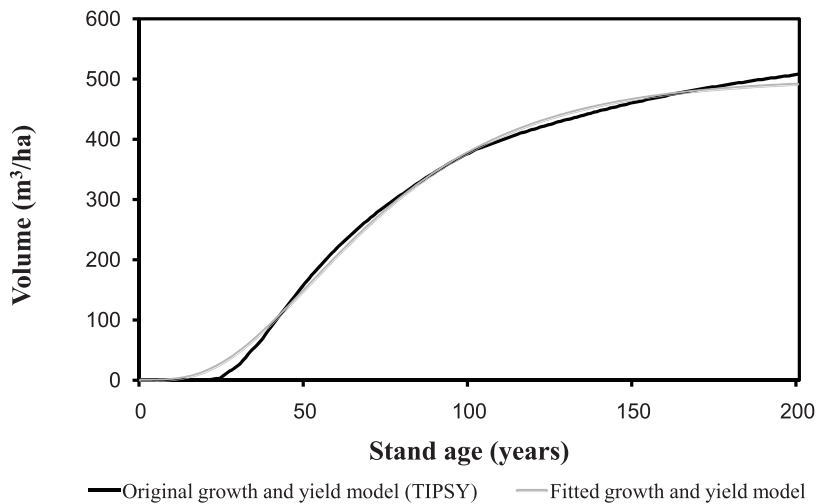


Figure 2. Comparison of TIPSY projection of merchantable stand yield with Chapman-Richards approximation

A residual value approach (Davis *et al.*, 2001, pp. 418–427) is used to estimate the stumpage value of timber. All costs and prices in this paper are expressed in Canadian dollars (CAD). The residual value is the selling price of the final products (in this case lumber and pulp chips produced as a byproduct) less the costs of converting standing trees into the final products, expressed in CAD/m^3 of merchantable timber.

The average lumber price of kiln dried, standard and better, western spruce-pine-fir, 2x4 random length lumber for the period April 1999 to March 2008 was 375 CAD/thousand board feet (MBF) (BC MoFR, 2009). Based on the observed range of lumber prices for this time period, low and high lumber prices (250 and 500 CAD/MBF) is used in sensitivity analyses. At the time of writing, lumber prices are very close to 250 CAD/MBF. The price of wood chips was assumed to be 70 CAD/bone dry unit (BDU). For the medium site pine stand at 80 years of age (the volume maximizing harvest age), TIPSY indicates that 0.210 MBF of lumber and 0.152 BDU of pulp chips will be produced per cubic metre of roundwood input. The base selling price of the final products expressed in equivalent roundwood input terms is 89.50 CAD/m³ (375 CAD/MBF 0.210 MBF/m³ + 70 CAD/BDU 0.152 BDU/m³ = 89.50 CAD/m³). The corresponding low and high estimates of selling price are 63.15 and 115.65 CAD/m³.

The cost of converting standing trees into end product is the sum of all costs associated with harvesting, hauling, and milling. Road construction and harvesting costs reported by TIPSY for the pine stand were 1,150, and 5,100 CAD/ha. Log hauling, milling and overhead costs as 4.84, 34.65, and 8.06 CAD/m³ respectively. Using the residual value approach and an average merchantable volume of 310 m³/ha at 80 years of age, the base, low, and high estimates of the value of standing timber are -4.46, 21.79, and 48.04 CAD/m³ respectively. The negative stand value is not unrealistic as evident by recent mill closures in Canada. This is partly because of the rising Canadian dollar and cheap lumber prices. In the short run it is possible that firms might operate even with negative stand value. Stands are assumed to be reestablished immediately following harvest at a cost of 1,250 CAD/ha. The costs reported on a CAD/ha basis are assumed to be closely related to the area harvested; the costs reported as CAD/m³ are assumed to be more

closely related to volume harvested.

2.1. Accounting for carbon sequestration

The decision problem is represented as a dynamic program, and state variables are used to describe the system at each stage of the decision problem. It is theoretically possible to develop a dynamic program with state variables representing carbon stock in each of the 20+ carbon pools represented in CBM-CFS3, but program solution becomes impractical due to Bellman's "curse of dimensionality" (Bellman, 1961). A simplified representation of the carbon pool structure of CBM-CFS3 is created using two carbon pools: a biomass pool representing carbon stored above and below ground in living trees, and a dead organic matter pool representing all other carbon stored in standing dead trees, on the forest floor, and in the soil. The label "dead organic matter" is used even though it is recognized that some of the carbon in this DOM pool is contained in living organisms.

Transfer of carbon from the biomass to the DOM pool is described by a litterfall rate expressed as a proportion of biomass, and transfer of carbon from the DOM pool to the atmosphere is described as a decay rate expressed as a proportion of DOM stocks. Under the approach used in this study, total ecosystem carbon sequestration, TEC_t measured in tonnes of carbon per hectare (tC/ha) at time t , is simply the sum of the carbon in the living biomass and the DOM pools.

$$[2] \quad TEC_t = B_t + DM_t$$

where B_t represents total carbon sequestered in the living biomass; and DM_t measures the total carbon sequestered in the DOM pool. The yield curve produced by TIPSYS was used as input to CBM-CFS3 to develop a projection of the stocks stored in the various carbon pools

for the lodgepole pine stand.

Figure 3 shows the development of aggregated carbon pool stocks for an unharvested lodgepole pine stand starting at age 0 as projected by CBM-CFS3. At time 0, there is no biomass and an initial stock of 370 tC/ha in the DOM pool. This is very near the (IPCC, 2000) estimate of average DOM carbon stocks of 340 tC/ha in the boreal forest. In the early stages of stand development, the stand is a net source of CO₂ as a result of decay processes (Kurz *et al.*, 1992). As the stand ages, TEC stocks increase with increasing biomass, and the decline in DOM stocks slows and reverses as carbon is added to the DOM pool in the form of litterfall, dead branches and natural tree mortality.

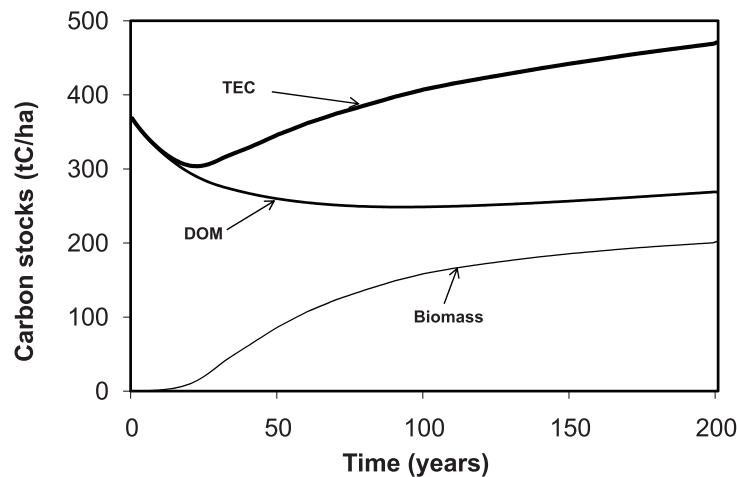


Figure 3. No timber harvest

Projection of aggregated carbon pool stocks from CBM-CFS3 for an unharvested lodgepole pine stand in the BWBS biogeoclimatic zone and in the Dawson Creek Forest District

Instead of using the detailed carbon pool representation in CBM-CFS3, the DOM carbon pools are aggregated into a single pool with

a single annual decay rate of α . The living biomass carbon pools were combined into a single carbon pool with an annual litter fall rate of β . The dynamics of the DOM pool are represented by [3].

$$[3] \quad DM_t = (1 - \alpha)DM_{t-1} + \beta B_{t-1}$$

where DM_t is the amount of DOM in period t and DM_{t-1} is the amount of DOM in the period $t - 1$. B_{t-1} is the carbon in living biomass in period $t - 1$. The decay and litterfall rates were estimated using the method of least squares to find the curve which best fits the DOM curve shown in Figure 4. The estimated parameters are $\alpha = 0.00841$ and $\beta = 0.01357$.

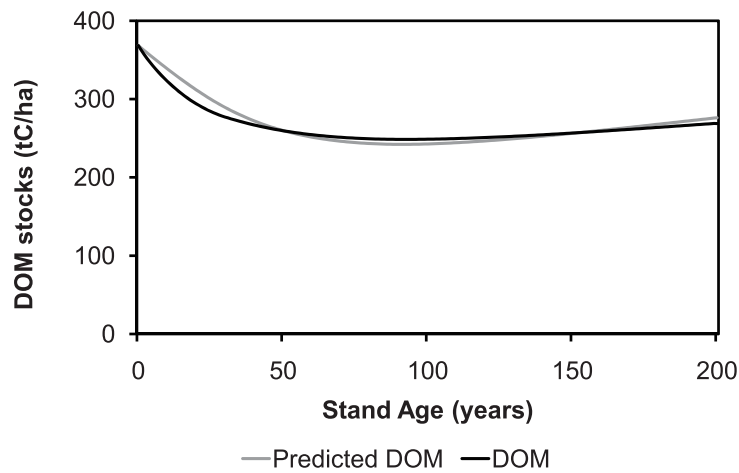


Figure 4. Comparison of DOM carbon stock projections using CBM-CFS3 and estimated approximation

Figure 4 compares the estimated DOM curve to that generated by CBM-CFS3. In general, the estimated curve corresponds well to the CBM-CFS projection, although it overestimates DOM carbon stocks

for stands younger than 50 years. This may be because the simplified model does not consider faster decay rates associated with younger, more open stands, and CBM-CFS3 does (Kurz *et al.*, 1992). However, this simplified representation is deemed adequate for the purposes of the current exposition.

When timber harvest occurs, the merchantable timber volume is removed from the site and processed into lumber and wood chips. The roots, stumps, tops, branches and leaves are assumed to die at the time of harvest and become part of the DOM pool. The difference between the pre-harvest stock of carbon in living biomass (B_t) and the carbon that is removed from the forest in the form of harvested wood (MC_t) is transferred to the DOM pool. Carbon removed from the forest is calculated as:

$$[4] \quad MC_t = \gamma H_t$$

where $\gamma = 0.2$ is the estimated average carbon content of wood (tC/m^3), and H_t is the volume removed at harvest (m^3). The carbon content of wood varies with tree species, although it is generally in the range of 200 kg m^{-3} (Jessome, 1977). In the absence of timber harvest, the amount of debris transferred D_t , assumes a value of zero. The amount of debris transferred to the DOM pool can generally be expressed as:

$$[5] \quad D_t = \begin{cases} 0 & : H_t = 0 \\ [B_t - \gamma H_t] & : H_t > 0 \end{cases}$$

Therefore, if the amount of debris transferred after harvest is taken into account, the amount of carbon in the DOM pool at any time can be recalibrated using [3]:

$$[6] \quad DM_t = (1 - \alpha)DM_{t-1} + \beta B_{t-1} + D_t$$

Figure 5, obtained through simulations, illustrates what happens to the DOM pool after a harvest. The figure shows that timber harvest in year 80 produces a sharp increase in the amount of DOM, followed by a period of net emission of CO₂ because the release from decomposing slash is greater than the carbon uptake in young trees.

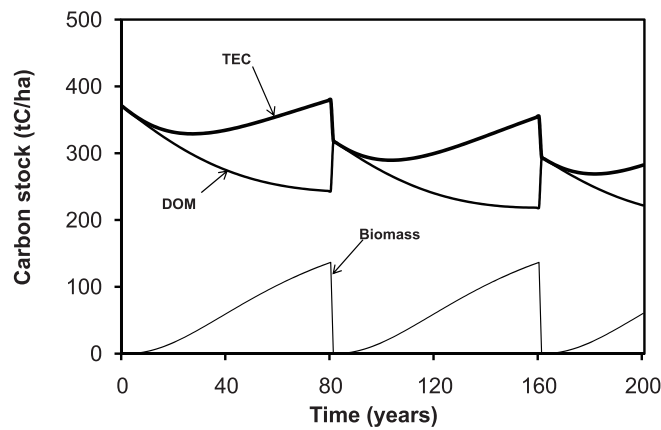


Figure 5. Timber harvest on an 80 year rotation

Projection of aggregated carbon pool stocks from CBM-CFS3 for an unharvested lodgepole pine stand in the BWBS biogioclimatic zone and in the Dawson Creek Forest District

2.2. Valuation of carbon

The carbon market assumed for this article pays landowners for net sequestration of CO₂ and requires payment for net release of CO₂ in the previous year. The price received per tonne of sequestered CO₂ is the same as the price paid per tonne of released CO₂. Net carbon sequestration is calculated as the change in total ecosystem carbon stocks:

$$[7] \quad \Delta TEC_t = TEC_t - TEC_{t-1}$$

In this study, a broad range of prices for CO₂ is used in sensitivity analyses. It is assumed the price of CO₂ (P^{CO_2}) to be 0, 1, 2, 5, 10, 20 or 50 CAD/t of CO₂ sequestered or released. The price of permanent carbon credits traded on European Climate Exchange (ECX) between January 2005 and April 2008 ranged from 10 to 45 CAD/tCO₂ (Point Carbon, 2009). Prices for carbon credits traded on the Chicago Climate Exchange (CCX) for the same time period ranged from 1 to 5 CAD/tCO₂ (CCX, 2009). The range of prices that have been chosen encompass the range of observed prices including any discounting that may occur in order to account for the temporary nature of carbon sequestration in forests. It is conventional to express carbon prices in currency units per tCO₂ and stocks as tonnes of carbon (tC). This practice is continued here for reporting, but for modeling purposes, an equivalent prices for carbon (CAD/tC) is defined as:

$$[8] \quad P^C = 3.67P^{CO_2}$$

because the molecular weight of CO₂ is approximately 3.67 times the atomic weight of carbon.

2.3. Dynamic Programming Model

The model developed here is a discrete backwards recursion dynamic programming model. The stages represent time, in one year time steps. The forest stand is described by a combination of two state variables, the age of the stand (years) and carbon stocks in the DOM pool (tC/ha). There are 251 discrete age classes, ranging from ages 0 to 250 years and 501 discrete DOM classes ranging from 0 to 500 tC/ha. The 250 year and 500 tC/ha classes are trapping classes that represent age 250 years and older and DOM carbon stocks of 499.5 tC/ha and greater. Timber harvest volume and carbon stored in the biomass pool

are calculated as a function of stand age.

At each point in time, a decision is made by the landowner whether to clearcut the stand or let the stand to grow for another year. Clearcutting yields immediate timber revenue. Both the clearcut and the leave decisions will result in a change in TEC and the appropriate carbon credit or debit. If harvesting does occur (i.e. decision, $h = 1$) in stage t , it is assumed that replanting occurs immediately and the stand age is reset to 1 in stage $t + 1$. If harvesting does not occur (i.e. decision, $h = 0$) in stage t , the stand age is incremented by one year in stage $t + 1$.

The stage return or periodic payoff (N_t) is calculated as shown in equation [9]. The payoff is calculated for the midpoints of each DOM class (d_i) and stand age (a_j) and for each of the possible harvest decisions (h). If a stand is not harvested ($h = 0$), the periodic payoff would be the carbon credit or debit occurring over a growth period. On the other hand if the stand is harvested ($h = 1$), a net harvest revenue is received from the sale of timber and either a debit is charged for net carbon loss or a credit is received for net carbon sequestered.

$$[9] \quad N_t\{d_i, a_j, h\} = \begin{cases} \delta PC [(1 - \alpha)d_i + \beta B(a_j) \\ \quad - d_i + B(a_{j+1}) - B(a_j)] & : h = 0 \\ \delta PC [(1 - \alpha)d_i + \beta B(a_j) \\ \quad - d_i + \beta B(a_1) \\ \quad - \gamma H(a_j)] + P^w H(a_j) & : h = 1 \end{cases}$$

where a_j = age of a stand in state j , and $j = 0, 1, \dots, 250$ years (with harvest, the age of the stand is set to 1 for the subsequent year, so the change in biomass becomes $B(a_1) - B(a_j)$); d_i = amount of DOM in state i , and $i = 0, 1, \dots, 500$ tC/ha; P^w = stumpage value of standing trees in CAD/m³; H = timber volume in m³/ha and E = regeneration cost in CAD/ha.

In this analysis, it is assumed that the objective is to determine, for each possible stand age, at each decision stage, the decision (harvest or do not harvest) that results in the maximum net present value of timber land for the joint production of timber and carbon sequestration over the planning horizon. The stages in this dynamic programming model correspond to the time periods in which decisions are made. It is a finite horizon, deterministic model with time t measured in years.

Because the objective function is to maximize net present value of timber and carbon, the discount rate becomes an important parameter. The discount factor δ shown in equation [10] represents the relative value of a dollar received one year from now (given an annual discount rate of i) to a dollar today. The discount rate used for the analysis is 5% per annum. This is the rate used in van Kooten *et al.* (1995) and is intended to reflect a market rate of time preference.

$$[10] \quad \delta = \frac{1}{(1+i)} = \frac{1}{1.05} = 0.9528$$

The return for the last stage in the problem is initialized to zero as defined in equation [11] :

$$[11] \quad R_T \{d_i, a_j\} = 0, \forall i, j$$

This assumption is justified on the basis that T is large (500 years) and the discounted value of R_T for reasonable discount rates for this problem is near zero (e.g. the present value of 1 CAD received 500 years in the future is 2.543×10^{-11} CAD) for a 5% discount rate.

The recursive objective function for this problem is given in equation [12]. The recursive function also includes the stage return function given in equation [9] and the appropriately discounted returns for each

of the possible harvest decisions:

$$[12] \quad R_t \{d_i, a_j\} = \left[\max_h N_t \{d_t, a_j, h\} = \begin{cases} \delta(\rho R_{t+1} [d_i, a_{j+1}]) & : h = 0 \\ \delta(\rho R_{t+1} [d_i, a_1]) & : h = 1 \end{cases} \right],$$

$t = T - 1, \dots, 2, 1.$

The recursive objective function defines the objective function for the stages $T - 1$ down to 1 for each possible combination of state variables. It calculates a return for each of the harvest decisions and selects the harvest decision that results in the maximum return as the optimal choice for the state combination in that stage.

Equation [13] below describes the stage return at time zero, R_0^* . This is done because there is the need to incorporate establishment costs at time zero, for stands of age 0. For subsequent harvests, establishments costs are incorporated in equation [9].

$$[13] \quad R_0^* \{d_i, 0\} = R_0 \{d_i, 0\} - E, \forall_i$$

Economic analyses for timber production management only were also conducted to produce a baseline for this study. The results from the different sets of runs are presented in the next section.

3. Results and Discussion

In order to characterize the behaviour of a landowner, a number of analyses have been undertaken. The main objective has been to analyze the sensitivity of optimal harvest decision to DOM stocks. Using a base lumber price of 375 CAD/MBF and a discount rate of 5%, attention was focused on four issues. First, the study tried to understand how alternative carbon prices will impact the optimal decision to harvest at different levels of DOM stocks. Second, the sensitivity of net present value to initial states was investigated. Third, a sensitivity analysis

based on different lumber prices was performed. Fourth, and finally, the effect of ignoring DOM in the model was investigated to allow for direct comparison with the work by van Kooten *et al.* (1995). The results presented in this section were calculated using an implementation of the dynamic programming model programmed in MATLAB (Pratap, 2006).

The main results of this study are presented in Figures 6–8. First the results of the case where carbon sequestration services have no value to the landowner (i.e. $P^{CO_2} = 0$) is discussed: the case of the Faustmann rotation. Figure 6 shows that the optimal harvest age for the base case scenario is 82 years. As carbon has no value, it is unsurprising that this harvest age is unrelated to the amount of carbon stored in the DOM pool.

In figure 7a, the value of land, standing timber and carbon sequestration services (LTCV, hereafter) is displayed for combinations of initial stand age and carbon stocks in the DOM pool. As expected, LTCV is unrelated to carbon stock when P^{CO_2} is zero, and increases with increasing age and, therefore, the volume of standing timber. Note that LTCV is negative below a stand age of 55 years. This is consistent with other results (Adamowicz *et al.*, 2003, Rodrigues, 1998) showing negative returns on timber growing investments in the boreal forest of Canada.

Figure 8a shows the development of the stand in state-space assuming the optimal decision rule is followed. Given the starting stock in the DOM pool of 370 tC/ha, an overall decline to a dynamic equilibrium is observed with about 275 tC/ha at stand establishment and 200 tC/ha at the optimal harvest age of 82 years. This equilibrium is essentially reached after 3 rotations.

Model runs were conducted using a wide range of carbon prices ($P^{CO_2} = 0, 1, 2, 5, 10, 20, 40$ CAD/tCO₂). In the interest of saving space,

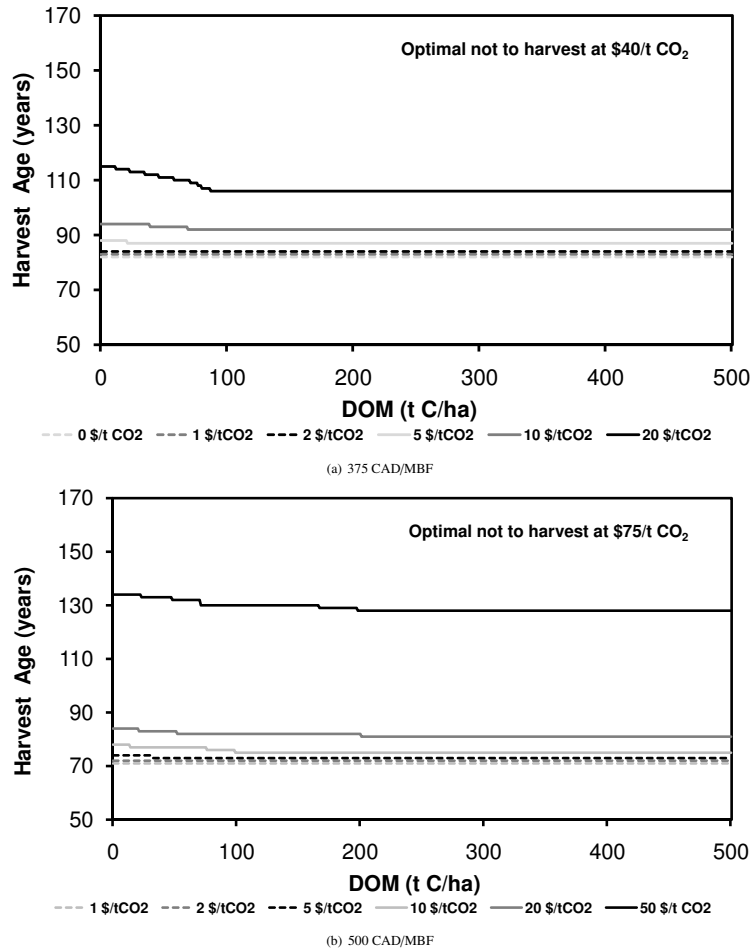


Figure 6. Optimal harvest age by DOM stocks and carbon price for alternative lumber prices

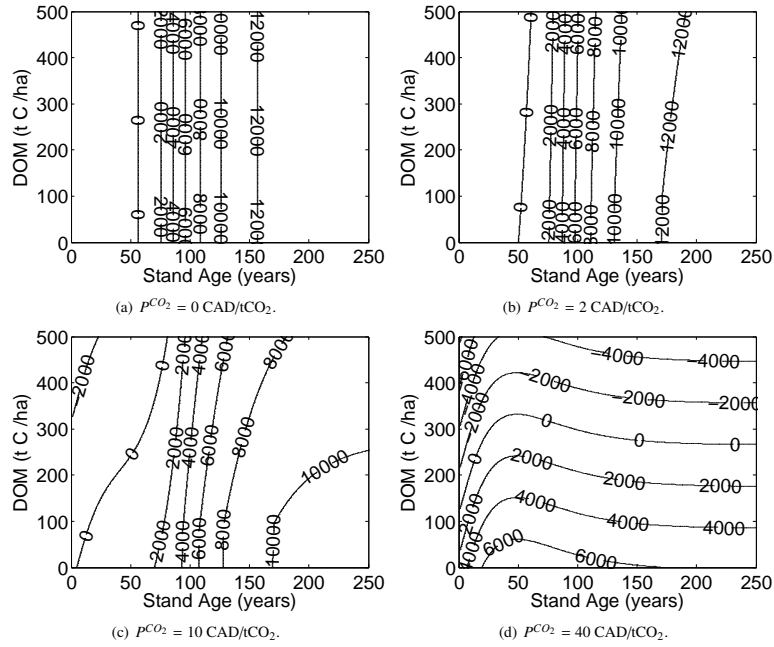


Figure 7. Land, timber and carbon values (CAD/ha) for different carbon prices

By stand age and carbon stocks in the DOM pool . The contours indicate combinations of age and DOM states that have the same land, timber and carbon values.

results for a subset of these prices are presented. In the next few paragraphs, Figures 6–8 examine the effects of carbon prices on the optimal harvest decision. From Figure 6 it is apparent that higher carbon prices are associated with older optimal harvest ages. In the base case scenario, the optimal harvest decision for carbon prices greater than 40 CAD/tCO₂ is to never harvest. The optimal policy is sensitive to DOM stocks at the lower levels. This happens because the amount of CO₂ released to the atmosphere through decomposition is lower with lower

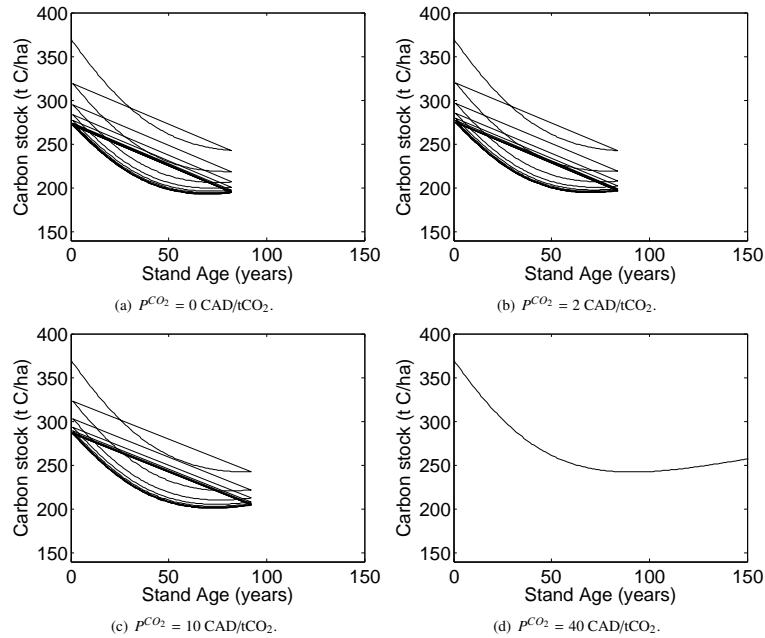


Figure 8. Stand development in state space given implementation of optimal decision rule

DOM stocks: the marginal gain in CO_2 sequestration from delaying harvest is greater with lower DOM stocks.

Figure 7 shows the combined values of land, timber, and carbon sequestrations services (LTCV) for different combinations of stand age, DOM stocks, and P^{CO_2} . Higher DOM carbon stocks lead to lower LTCVs because a fixed decay rate leads to more CO_2 being released from a large stock of DOM than a small stock. As carbon prices increase, there is an increase in LTCV for stands with a low DOM carbon stock, and a decrease in LTCV for stands with a high DOM carbon stock. For the examples presented here, the boundary between low and high DOM carbon stock stands is roughly between 100 and 250 tC/ha,

well below the boreal forest average of 340 tC/ha.

Figure 8 shows the trajectory of the stand in state-space given implementation of the optimal decision rule determined for alternative P^{CO_2} . In general, the optimal harvest age increases with increasing P^{CO_2} and reaches a point where the optimal harvest decision is to never harvest. At equilibrium, the DOM carbon stocks are higher with higher P^{CO_2} , but the difference is not substantial until the price is high enough for a never harvest optimum.

For completeness sake, sensitivity analysis was conducted using alternative lumber prices. Figure 6 presents the optimal harvest age by DOM carbon stocks and P^{CO_2} for lumber prices of 375, and 500 CAD/MBF. As the lumber price increases, the optimal harvest age shortens. This is consistent with previous optimal rotation analysis, and is unsurprising. At the low lumber price (i.e., 250 CAD/MBF-not shown), the never harvest decision becomes optimal for P^{CO_2} of 5 CAD/tCO₂ or greater; for the high lumber price, the never harvest decision is optimal for P^{CO_2} of 75 CAD/tCO₂ or greater.

A major difference between this study and those of van Kooten *et al.* (1995), Spring *et al.* (2005a,b), Chladná (2007), and Yoshimoto and Marušák (2007) is that this study considers carbon stored in biomass and DOM pools, whereas they ignore biomass pool. In order to evaluate the effect of ignoring the DOM pool, a series of model runs were conducted with a modified version of this model where the carbon market considered only biomass carbon. The results are summarized in Table 1.

In general, the optimal harvest age is older when the DOM pool is ignored. The older harvest age is probably related to the fact that the amount of carbon released to the atmosphere through decay is greater when the stock of carbon in DOM is considered. The optimizing model will choose a younger harvest age in order to minimize the financial

Table 1. Summary of harvest ages for “with” or “without” DOM pool.

P^{CO_2} (CAD/tCO ₂)	Rotation age	
	with DOM (years)	without DOM (years)
0	82	82
1	84	85
2	85	86
5	88	91
10	94	102
20	107	150
40	∞	∞

penalties associated with carbon loss through decay.

4. Conclusions

This study presents the formulation of, and results from, a dynamic programming model used to determine the optimal harvest decision for a forest stand used to provide both timber harvest volume and carbon sequestration services. The forest stand is described using two state variables: stand age and the stocks of carbon stored in the DOM pool. This study is unique in that it examines the impact of varying DOM on the optimal harvest age. This study provides a basic framework for assessing the economic implications of alternative methods of accounting for carbon stocks in DOM.

The model is used to examine optimal harvest decisions for a lodgepole pine stand in the boreal forest of western Canada. The following main conclusions can be drawn from the study:

1. Optimal harvest age increases with increasing carbon price,
2. The optimal decision is sensitive to the stocks of carbon in the DOM pool, especially when carbon prices are high and initial DOM stocks are low.

3. The optimal harvest age is younger when the dynamics of carbon in DOM are considered than when they are ignored.
4. In a market where a forest landowner is required to pay for CO₂ emissions from the forest, the carbon market may well reduce the value of forest from the case where timber values only are considered. In the analysis, this is true when stocks of carbon in the DOM pool are greater than about 200 tC/ha at the time of stand initiation. This level is about half that of the average carbon stocks in the boreal forest of Canada. The results suggests that in the boreal forest a landowner will not voluntarily participate in this type of carbon market if the starting DOM is greater than 200 tC/ha.

This article presented the results of an optimal harvesting model for a forest stand where the landowner is paid for net increases in total ecosystem carbon in the stand, and pays for net decreases, on an annual basis. By approximating a detailed carbon budget simulation model using two carbon pools, a dynamic programming model of the system is developed to capture the important elements of the system for an economic analysis. A variants of this model will be developed in the near future to explore alternative forms of carbon markets, including one which accounts for carbon pools in forest products.

5. Acknowledgements

I will like to express my sincere gratitude to Tohoku University Ecosystem Adaptability of Global COE and the Institute of Statistical Mathematics for providing financial support for me to present this research at the International Symposium, FORMATH in Tachikawea in March 2010. I will also like to acknowledge the contributions of the members of the editorial board, Dr. Atsushi Yoshimoto and Dr.

Masashi Konoshima as well as the anonymous reviewer whose comments greatly improved this article. Finally, I will like to acknowledge Dr. Glen W. Armstrong for the initial idea and for being a co-author for a different version of this article.

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