隣接条件下での森林資源管理

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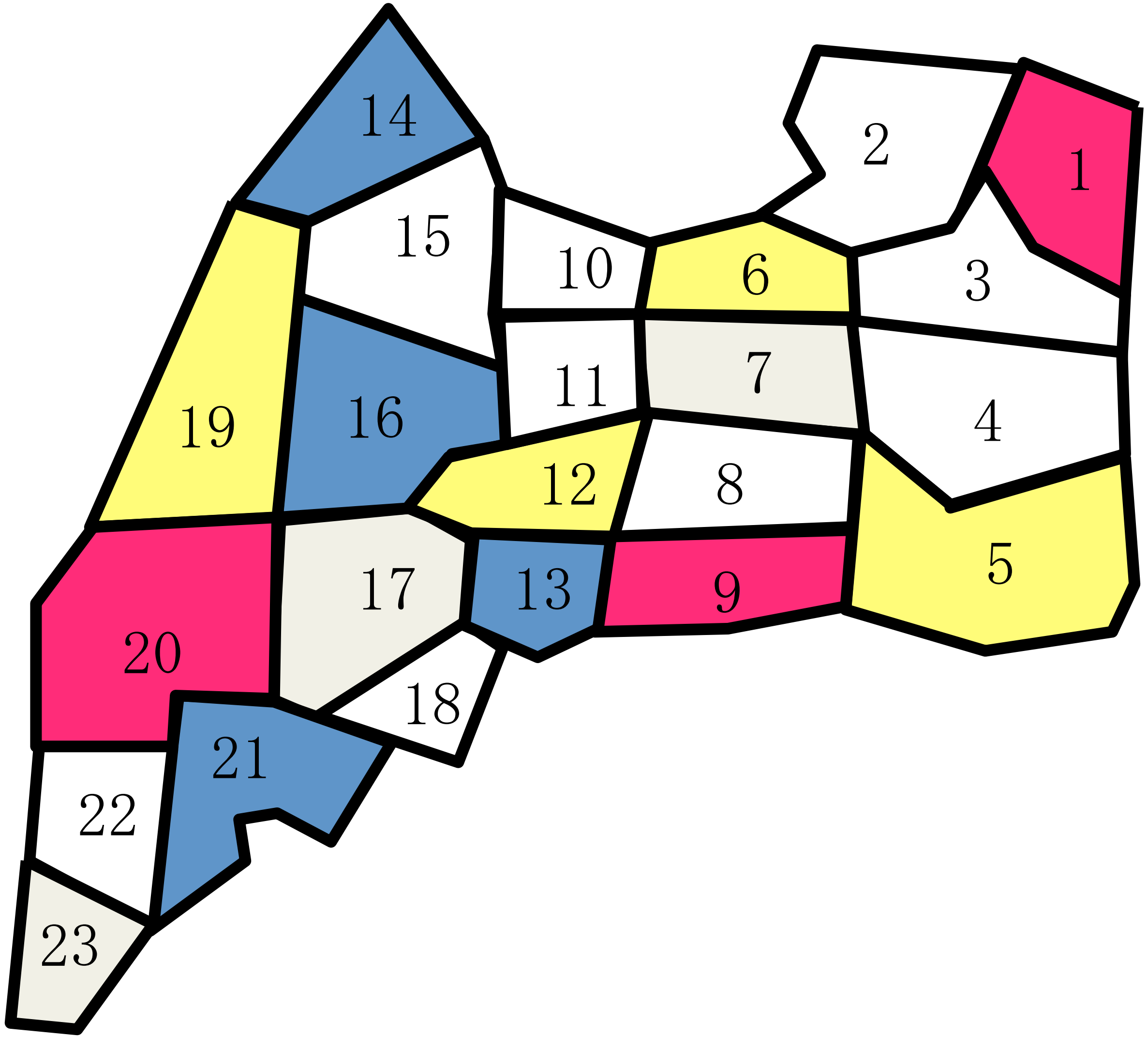
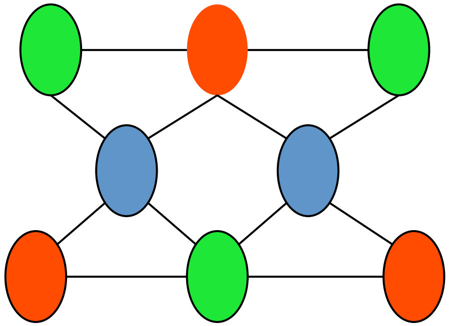
1. 分断化：Fragmentation



隣接の時代

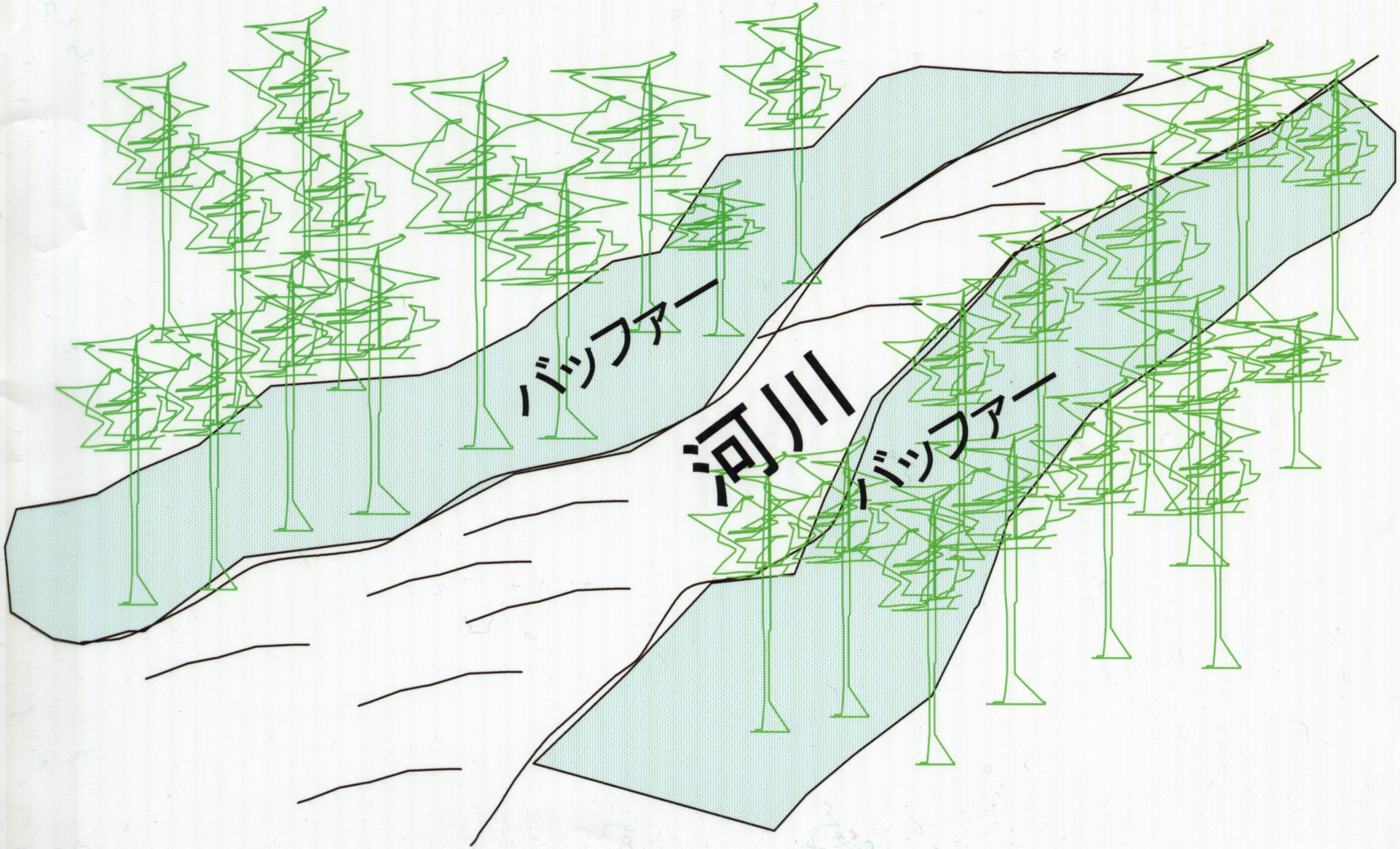


隣接林分での同時施業の回避：分断（Fragmentation)

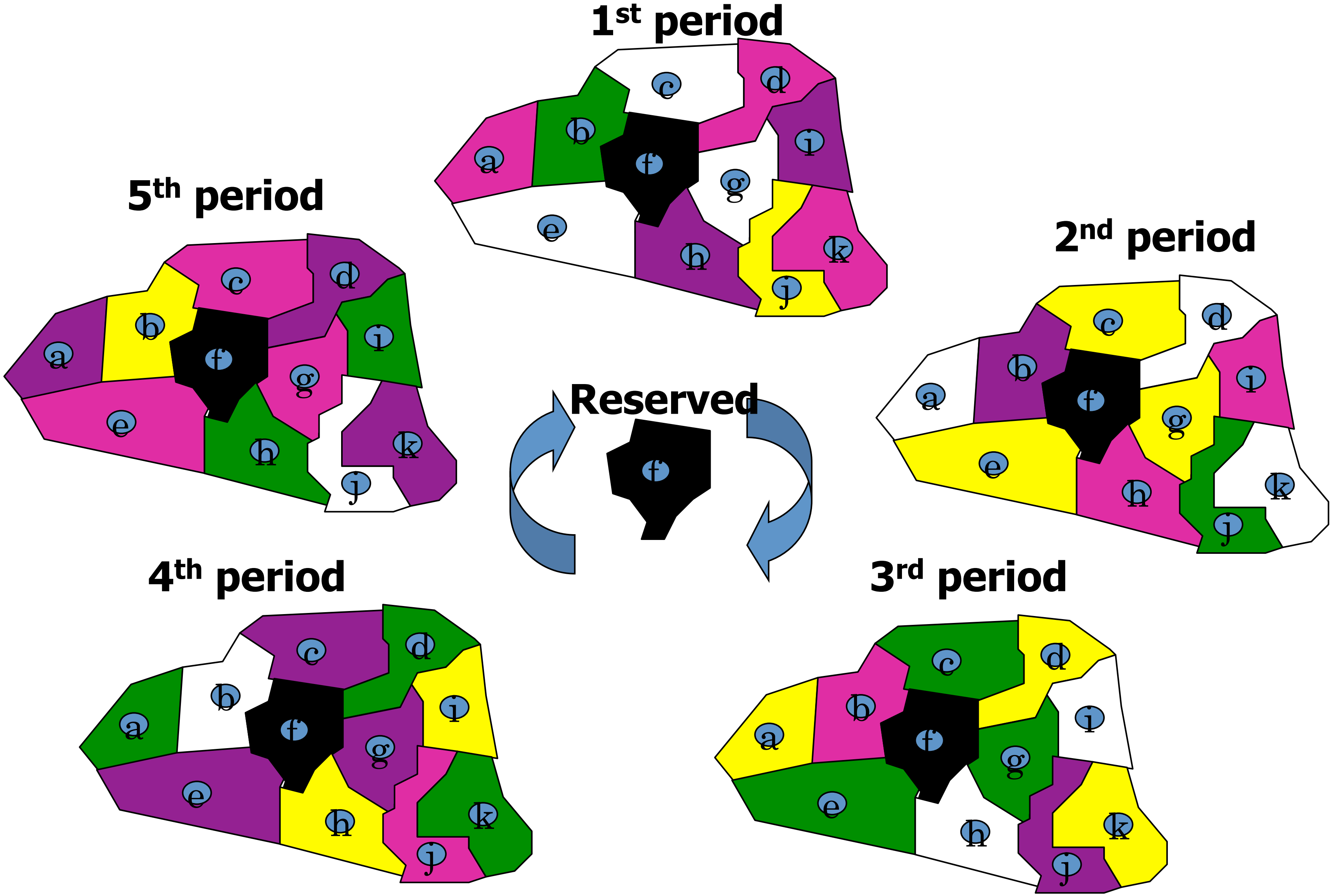
例: 林縁効果

Green-up Constraints

陸水生態系維持のための流域管理



制約の評価


1st period
5th period
2nd period
Reserved
4th period
3rd period
a
b
c
d
e
f
g
h
i
j
k

Definition of variables & others with Single Harvest

Decision variable matrix

$$\boldsymbol{X} = \begin{pmatrix} x_{1,1} & \cdots & x_{1,T} \\ \vdots & \ddots & \vdots \\ x_{m,1} & \cdots & x_{m,T} \end{pmatrix}, \qquad x_{i,j} = \begin{cases} 1 & \text{if the } i\text{-th unit is harvested at period } j \\ 0 & \text{otherwise} \end{cases}$$

Coefficient matrix

$$\boldsymbol{C} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,T} \\ \vdots & \ddots & \vdots \\ c_{m,1} & \cdots & c_{m,T} \end{pmatrix} \qquad c_{i,j} \text{ : coefficient of } x_{i,j}$$

Volume flow matrix

$$\boldsymbol{V} = \begin{pmatrix} v_{1,1} & \cdots & v_{1,T} \\ \vdots & \ddots & \vdots \\ v_{1,m} & \cdots & v_{m,T} \end{pmatrix} \qquad v_{i,j} \text{ : volume flow from } x_{i.j} \text{ at period } j$$

0-1 Integer Programming Formulation

Max PNV

$$Z = \max_{\mathbf{X}} \operatorname{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m}\sum_{j=1}^{T} c_{i,j} \cdot x_{i,j}$$

$st.$

$\boldsymbol{X}\mathbf{1}_T \leq \mathbf{1}_m$ Land Accounting Constraints

$(1-\alpha)v_0 \leq \boldsymbol{v}'^{(j)}\boldsymbol{x}^{(j)} \leq (1+\alpha)v_0, \qquad j = 1, \cdots, T$

where Harvest Flow Constraints

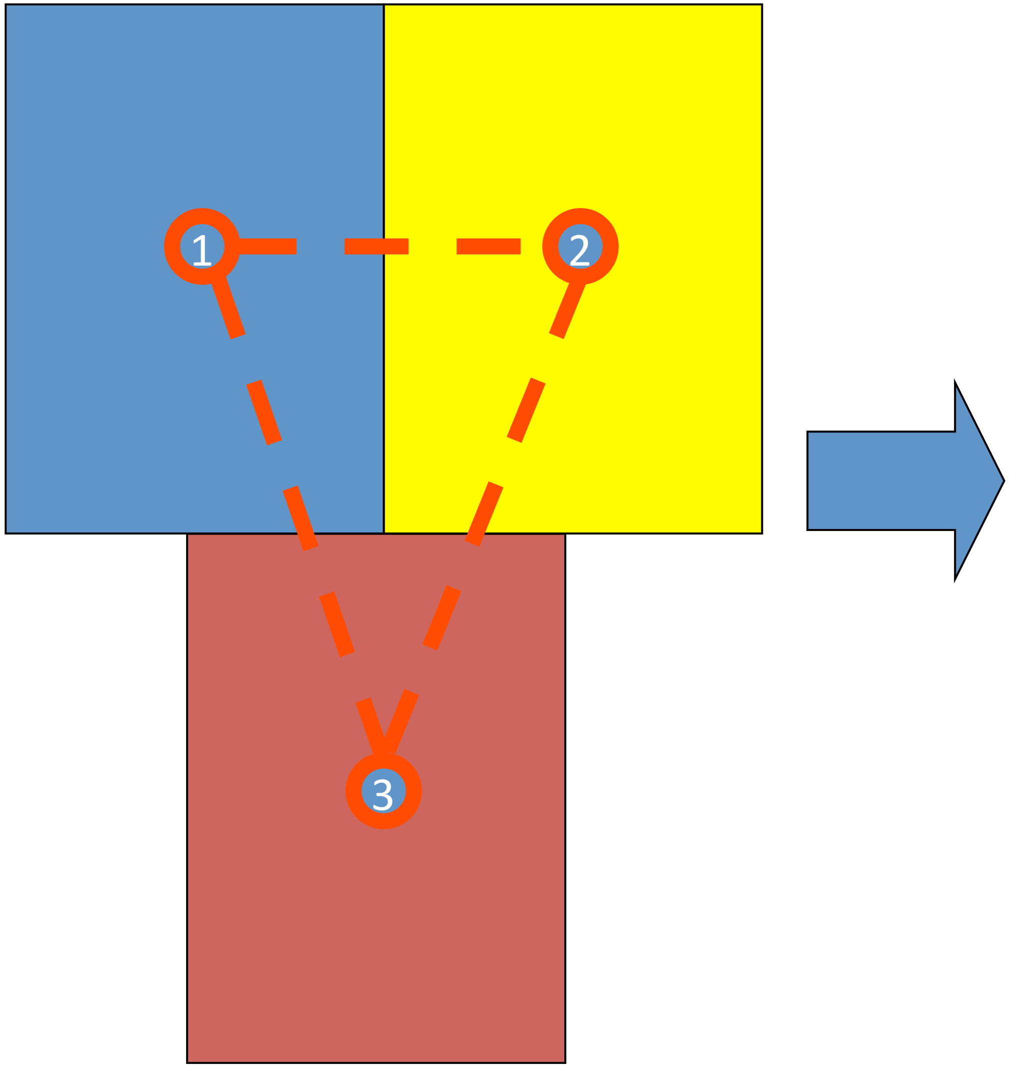
$\boldsymbol{v}^{(j)}$ and $\boldsymbol{x}^{(j)}$ are the j-th column vector of $\boldsymbol{V}$ and $\boldsymbol{X}$, respectively

+ Adjacency Constraints

Note: trace of a square matrix is the summation of diagonal elements

$$A = \begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix} \qquad \Rightarrow \qquad \operatorname{tr}(A) = \sum_{i=1}^{n} a_{i,i}$$

Adjacent Relationship



Adjacency List

1,2,3

2,1,3

3,1,2

Adjacency Matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Adjacency Constraints

Adjacency Matrix → Adjacency Constraints

Yoshimoto & Brodie (1994) Can J. For Res

Define:

$\boldsymbol{x}^{(j)}$: a j-th column vector of $\boldsymbol{X}$

$\boldsymbol{A}$: an $(m \times m)$ adjacency matrix

NB_i : a set of units adjacent to the i-th unit

Setting the following:

$\boldsymbol{m}_0 = \boldsymbol{A} \cdot \mathbf{1}_m$: summation of row elements of $\boldsymbol{A}$

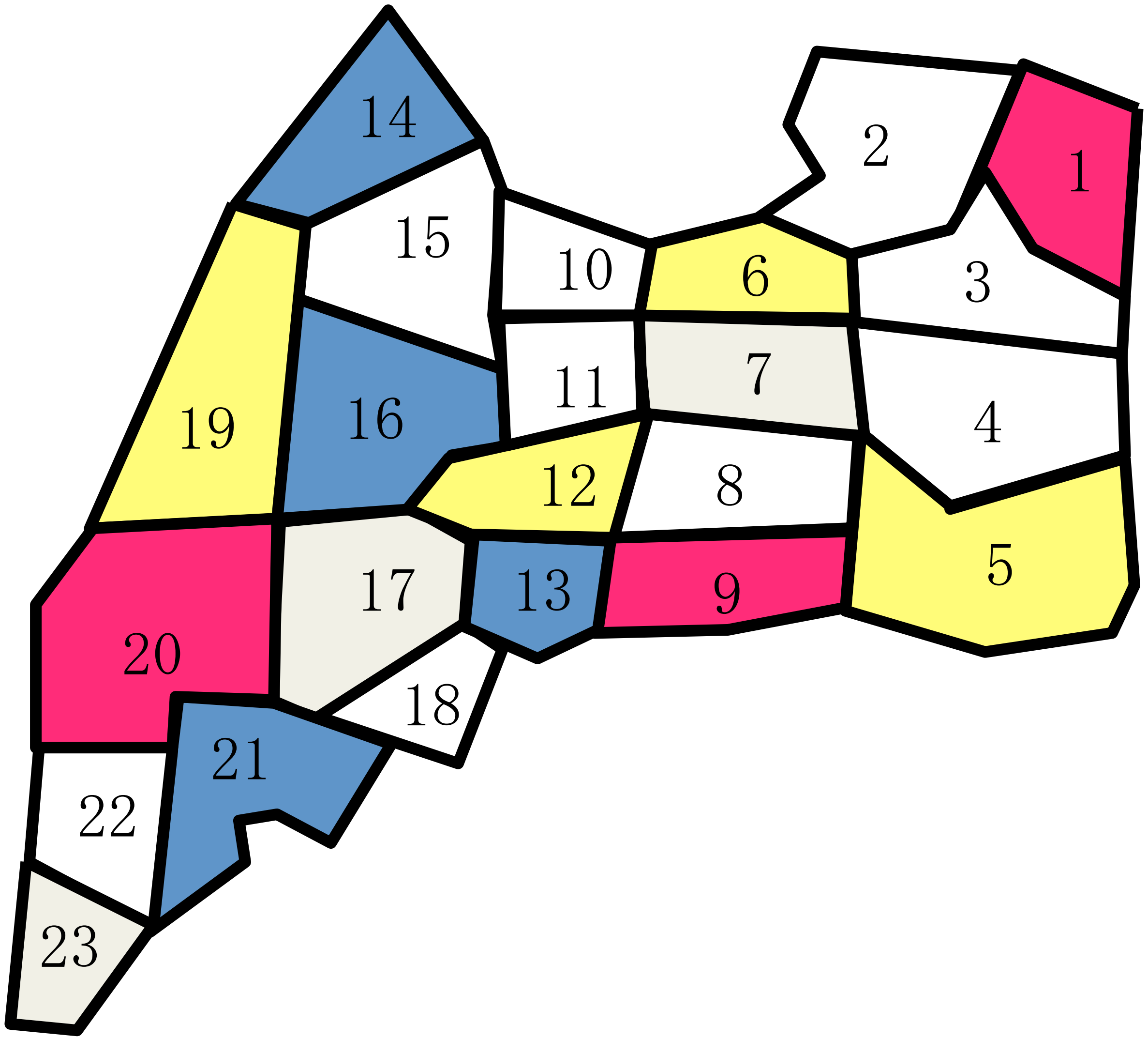
$\boldsymbol{M} = \boldsymbol{A} + \mathrm{diag}(\boldsymbol{m}_0)$

$$a_{i,j} = \begin{cases} 1 & \text{if } j \in NB_i \\ 0 & \text{if } j \notin NB_i \end{cases}$$

$$\boxed{\boldsymbol{M} \cdot \boldsymbol{x}^{(j)} \leq \boldsymbol{m}_0 \qquad \text{for } j = 1, \ldots, T}$$

Spatial Constraints

Spatial Adjacency Constraints



$$
\boldsymbol{m}_0 = \begin{pmatrix} 2 \\ 3 \\ 5 \\ 4 \\ 3 \\ 3 \\ 3 \\ 3 \\ 2 \end{pmatrix}, \qquad \boldsymbol{M} = \begin{pmatrix} 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 5 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 2 \end{pmatrix}
$$

$$
\boldsymbol{M} \cdot \boldsymbol{x}^{(j)} \leq \boldsymbol{m}_0
$$

$$
\boxed{[\boldsymbol{A} + \mathrm{diag}(\boldsymbol{A} \cdot \mathbf{1}_m)] \cdot \boldsymbol{x}^{(j)} \leq \boldsymbol{A} \cdot \mathbf{1}_m}
$$

Adjacency Problem with Single Harvest

Max PNV

$$Z = \max_{\mathbf{X}} \operatorname{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m}\sum_{j=1}^{T} c_{i,j} \cdot x_{i,j}$$

$$st.$$

$$\boldsymbol{X}\mathbf{1}_T \le \mathbf{1}_m$$

Land Accounting Constraints

$$(1-\alpha)v_0 \le \boldsymbol{v}'^{(j)}\boldsymbol{x}^{(j)} \le (1+\alpha)v_0, \qquad j = 1,\cdots,T$$

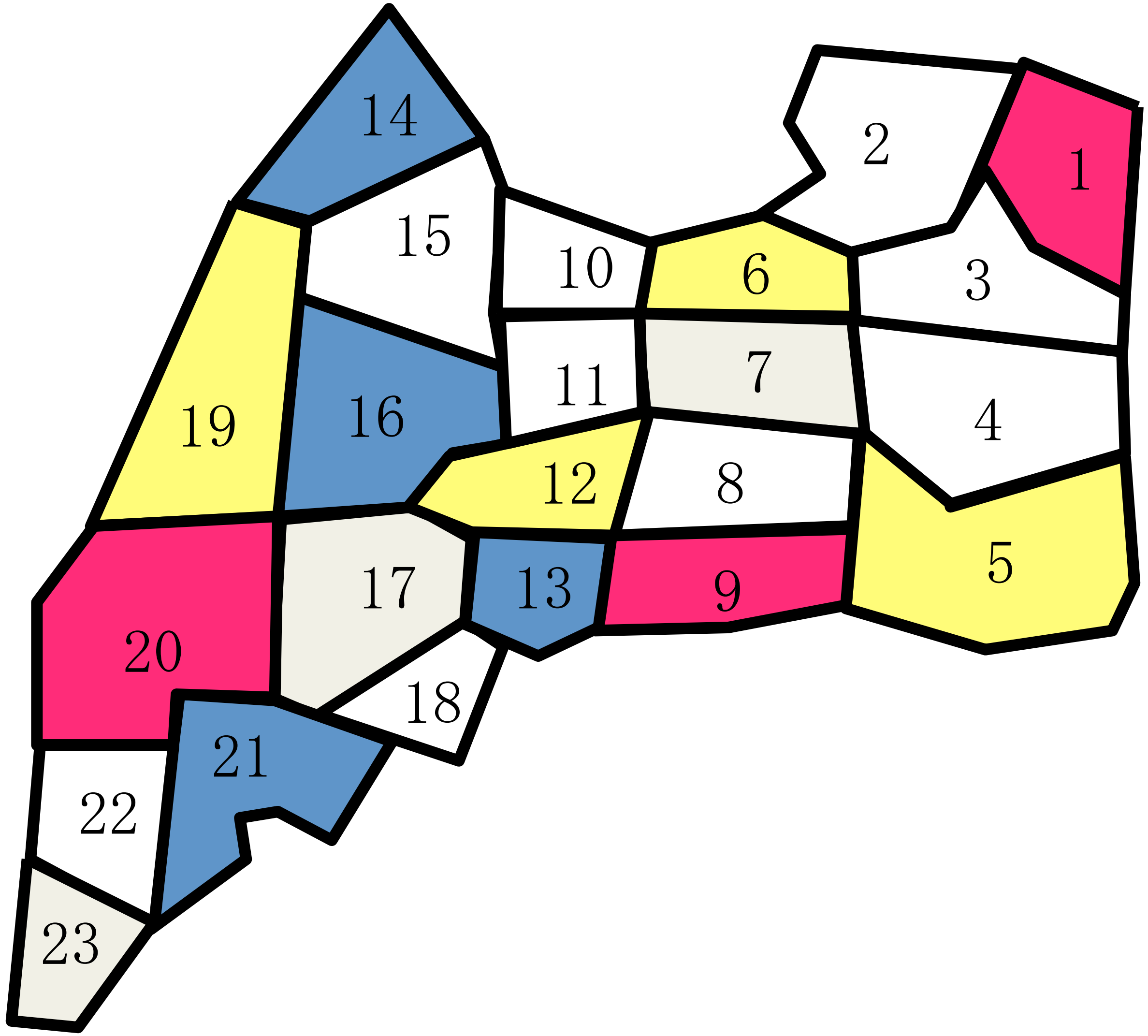
Harvest Flow Constraints

$$[\boldsymbol{A} + \operatorname{diag}(\boldsymbol{A}\cdot\mathbf{1}_m)]\cdot\boldsymbol{x}^{(j)} \le \boldsymbol{A}\cdot\mathbf{1}_m, \qquad \forall j$$

Adjacency Constraints

$$x_{i,j} = \begin{cases} 1 & \text{if the } i\text{-th unit is harvested at } j\text{-th period} \\ 0 & \text{otherwise} \end{cases}$$

Spatial Adjacency Problem with Multiple Harvests



Definition of variables & others

Decision variable matrix

$$\boldsymbol{X} = \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \ddots & \vdots \\ x_{m,1} & \cdots & x_{m,n} \end{pmatrix}$$

$x_{i,j}$: portion of the j-th treatment is implemented for the i-th unit

Coefficient matrix

$$\boldsymbol{C} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,n} \\ \vdots & \ddots & \vdots \\ c_{m,1} & \cdots & c_{m,n} \end{pmatrix}$$

$c_{i,j}$: coefficient of $x_{i,j}$

Volume flow matrix at period p

$$\boldsymbol{V}_p = \begin{pmatrix} v^p_{1,1} & \cdots & v^p_{1,n} \\ \vdots & \ddots & \vdots \\ v^p_{1,m} & \cdots & v^p_{m,n} \end{pmatrix}$$

$v^p_{i,j}$: volume flow from $x_{i.j}$

0-1 Integer Programming Formulation

Max PNV

$$Z = \max_{\mathbf{X}} \operatorname{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{i,j} \cdot x_{i,j}$$

$st.$

$\boldsymbol{X}\mathbf{1}_n \leq \mathbf{1}_m$ Land Accounting Constraints

$$(1-\alpha)v_0 \leq \operatorname{tr}(\boldsymbol{V}_p'\boldsymbol{X}) \leq (1+\alpha)v_0, \qquad p = 1,\cdots,T$$

Harvest Flow Constraints

$$x_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th treatment is implemented for the } i\text{-th unit} \\ 0 & \text{otherwise} \end{cases}$$

+ Adjacency Constraints

Adjacency Constraints for Model I type Decision Variables

Introduce:

Two types of adjacency for a decision variable matrix

1. **Spatial Adjacency** to consider spatial allocation of forest units $\boxed{\boldsymbol{A}^S}$

2. **Activity Adjacency** to consider concurrent treatments $\boxed{\boldsymbol{A}^V}$

Spatial Adjacency Matrix

$$\boldsymbol{A}^S : (m \times m) \quad \text{where} \quad a^s_{i,j} = \begin{cases} 1 & \text{if the } i\text{-th and } j\text{-th units are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{A}^S = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Activity Adjacency

Minimum Rotation: 6 periods

Table 1: Example of treatments

Treatment No.	Decision Variable	Coefficient	Period 1	2	3	4	5	6	7	8	9	10
1	$x_{i,1}$	$c_{i,1}$	X	0	0	0	0	0	0	0	0	0
2	$x_{i,2}$	$c_{i,2}$	X	0	0	0	0	0	X	0	0	0
3	$x_{i,3}$	$c_{i,3}$	X	0	0	0	0	0	0	X	0	0
4	$x_{i,4}$	$c_{i,4}$	X	0	0	0	0	0	0	0	X	0
5	$x_{i,5}$	$c_{i,5}$	X	0	0	0	0	0	0	0	0	X
6	$x_{i,6}$	$c_{i,6}$	0	X	0	0	0	0	0	0	0	0
7	$x_{i,7}$	$c_{i,7}$	0	X	0	0	0	0	0	X	0	0
8	$x_{i,8}$	$c_{i,8}$	0	X	0	0	0	0	0	0	X	0
9	$x_{i,9}$	$c_{i,9}$	0	X	0	0	0	0	0	0	0	X
10	$x_{i,10}$	$c_{i,10}$	0	0	X	0	0	0	0	0	0	0
11	$x_{i,11}$	$c_{i,11}$	0	0	X	0	0	0	0	0	X	0
12	$x_{i,12}$	$c_{i,12}$	0	0	X	0	0	0	0	0	0	X
13	$x_{i,13}$	$c_{i,13}$	0	0	0	X	0	0	0	0	0	0
14	$x_{i,14}$	$c_{i,14}$	0	0	0	X	0	0	0	0	0	X
15	$x_{i,15}$	$c_{i,15}$	0	0	0	0	X	0	0	0	0	0
16	$x_{i,16}$	$c_{i,16}$	0	0	0	0	0	X	0	0	0	0
17	$x_{i,17}$	$c_{i,17}$	0	0	0	0	0	0	X	0	0	0
18	$x_{i,18}$	$c_{i,18}$	0	0	0	0	0	0	0	X	0	0
19	$x_{i,19}$	$c_{i,19}$	0	0	0	0	0	0	0	0	X	0
20	$x_{i,20}$	$c_{i,20}$	0	0	0	0	0	0	0	0	0	X

Treatment

Note: X denotes harvesting while 0 denotes no harvesting

Activity Adjacency Matrix

$$\boldsymbol{A}^{V} : (n \times n) \text{ where } a^{v}_{i,j} = \begin{cases} 1 & \text{if the } i\text{-th and } j\text{-th treatments are concurrent} \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{A}^{V} = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Adjacency Matrix for a decision variable matrix to avoid harvests at adjacent units

Vectorization of a decision matrix to a decision vector

$$\mathrm{vec}(\boldsymbol{X}') = (\underbrace{x_{1,1}, \cdots, x_{1,n}}_{1^{\text{st}}\text{ unit}}, \underbrace{x_{2,1}, \cdots, x_{2,n}}_{2^{\text{nd}}\text{ unit}}, \cdots, \underbrace{x_{m,1}, x_{m,2}, \cdots, x_{m,n}}_{m^{\text{th}}\text{ unit}})'$$

Adjacency Matrix for the above decision vector becomes Kronecker product of spatial and activity adjacency matrices

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}^S \otimes \boldsymbol{A}^V = \begin{pmatrix} a^s_{1,1}\boldsymbol{A}^V & \cdots & a^s_{1,m}\boldsymbol{A}^V \\ \vdots & \ddots & \vdots \\ a^s_{m,1}\boldsymbol{A}^V & \cdots & a^s_{m,m}\boldsymbol{A}^V \end{pmatrix} \quad (mn \times mn)$$

where

$\otimes$: Kronecker product

隣接制約定式化
Model I type decision variables

Max PNV

$$Z = \max_{\boldsymbol{X}} \operatorname{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{i,j} \cdot x_{i,j}$$

$st.$

$$\boldsymbol{X}\mathbf{1}_n = \mathbf{1}_m$$

Land Accounting Constraints

$$(1-\alpha)v_0 \le \operatorname{tr}(\boldsymbol{V}_p'\boldsymbol{X}) \le (1+\alpha)v_0, \qquad p = 1, 2, \cdots, T$$

Harvest Flow Constraints

$$\widetilde{\boldsymbol{A}} + \operatorname{diag}(\boldsymbol{m}_0) \cdot \operatorname{vec}(\boldsymbol{X}') \le \widetilde{\boldsymbol{A}} \cdot \mathbf{1}_{mn}$$

$$\widetilde{\boldsymbol{A}} = \boldsymbol{A}^S \otimes \boldsymbol{A}^V$$

Adjacency Constraints

$$x_{i,j} = \begin{cases} 1 & \text{if } j\text{-th treatment is implemented for the } i\text{-th stand} \\ 0 & \text{otherwise} \end{cases}$$

Green-up constraints avoid adjacent harvests over the green-up period

Green-up Problem：植生回復期間制約

Green-up constraints avoid adjacent harvests over the green-up period

Adjacency constraints hold over the green-up period
Modify Activity Adjacent Matrix to avoid the green-up

Activity Adjacency

Table 1: Example of treatments

Minimum Rotation: 6 periods

Treatment No.	Decision Variable	Coefficient	Period 1	2	3	4	5	6	7	8	9	10
1	$x_{i,1}$	$c_{i,1}$	X	0	0	0	0	0	0	0	0	0
2	$x_{i,2}$	$c_{i,2}$	X	0	0	0	0	0	X	0	0	0
3	$x_{i,3}$	$c_{i,3}$	X	0	0	0	0	0	0	X	0	0
4	$x_{i,4}$	$c_{i,4}$	X	0	0	0	0	0	0	0	X	0
5	$x_{i,5}$	$c_{i,5}$	X	0	0	0	0	0	0	0	0	X
6	$x_{i,6}$	$c_{i,6}$	0	X	0	0	0	0	0	0	0	0
7	$x_{i,7}$	$c_{i,7}$	0	X	0	0	0	0	0	X	0	0
8	$x_{i,8}$	$c_{i,8}$	0	X	0	0	0	0	0	0	X	0
9	$x_{i,9}$	$c_{i,9}$	0	X	0	0	0	0	0	0	0	X
10	$x_{i,10}$	$c_{i,10}$	0	0	X	0	0	0	0	0	0	0
11	$x_{i,11}$	$c_{i,11}$	0	0	X	0	0	0	0	0	X	0
12	$x_{i,12}$	$c_{i,12}$	0	0	X	0	0	0	0	0	0	X
13	$x_{i,13}$	$c_{i,13}$	0	0	0	X	0	0	0	0	0	0
14	$x_{i,14}$	$c_{i,14}$	0	0	0	X	0	0	0	0	0	X
15	$x_{i,15}$	$c_{i,15}$	0	0	0	0	X	0	0	0	0	0
16	$x_{i,16}$	$c_{i,16}$	0	0	0	0	0	X	0	0	0	0
17	$x_{i,17}$	$c_{i,17}$	0	0	0	0	0	0	X	0	0	0
18	$x_{i,18}$	$c_{i,18}$	0	0	0	0	0	0	0	X	0	0
19	$x_{i,19}$	$c_{i,19}$	0	0	0	0	0	0	0	0	X	0
20	$x_{i,20}$	$c_{i,20}$	0	0	0	0	0	0	0	0	0	X

Treatment

Note: X denotes harvesting while 0 denotes no harvesting

Activity Adjacency Matrix

$$\boldsymbol{A}_1^V : (n \times n) \text{ where } a_{i,j}^v = \begin{cases} 1 & \text{if the } i\text{-th and } j\text{-th treatments are concurrent over 1 period} \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{A}_1^V = \begin{pmatrix}
1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
1&1&1&1&1&0&1&0&0&0&0&0&0&0&0&0&0&1&0&0\\
1&1&1&1&1&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0\\
1&1&1&1&1&0&0&0&1&0&0&1&0&1&0&0&0&0&0&1\\
0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&1&1&1&1&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&1&0&1&1&1&1&0&1&0&0&0&0&0&0&0&1&0\\
0&0&0&0&1&1&1&1&1&0&0&1&0&1&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&1&0&1&1&1&0&0&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&1&1&1&1&0&1&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&1&0&0&1&1&1&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&1&0&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&1&0&0&1&0&1&0&0&0&0&0&1
\end{pmatrix}$$

1 Green-up

Adjacency Matrix

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}^S \otimes \boldsymbol{A}_1^V = \begin{pmatrix} a_{1,1}^s \boldsymbol{A}_1^V & \cdots & a_{1,m}^s \boldsymbol{A}_1^V \\ \vdots & \ddots & \vdots \\ a_{m,1}^s \boldsymbol{A}_1^V & \cdots & a_{m,m}^s \boldsymbol{A}_1^V \end{pmatrix} \quad (mn \times mn)$$

where

$\otimes$: Kronecker product

Activity Adjacency for Green-Up

Table 1: Example of treatments

Minimum Rotation: 6 periods

2 Green-up

Treatment No.	Decision Variable	Coefficient	Period 1	2	3	4	5	6	7	8	9	10
1	$x_{i,1}$	$c_{i,1}$	X	0	0	0	0	0	0	0	0	0
2	$x_{i,2}$	$c_{i,2}$	X	0	0	0	0	0	X	0	0	0
3	$x_{i,3}$	$c_{i,3}$	X	0	0	0	0	0	0	X	0	0
4	$x_{i,4}$	$c_{i,4}$	X	0	0	0	0	0	0	0	X	0
5	$x_{i,5}$	$c_{i,5}$	X	0	0	0	0	0	0	0	0	X
6	$x_{i,6}$	$c_{i,6}$	0	X	0	0	0	0	0	0	0	0
7	$x_{i,7}$	$c_{i,7}$	0	X	0	0	0	0	0	X	0	0
8	$x_{i,8}$	$c_{i,8}$	0	X	0	0	0	0	0	0	X	0
9	$x_{i,9}$	$c_{i,9}$	0	X	0	0	0	0	0	0	0	X
10	$x_{i,10}$	$c_{i,10}$	0	0	X	0	0	0	0	0	0	0
11	$x_{i,11}$	$c_{i,11}$	0	0	X	0	0	0	0	0	X	0
12	$x_{i,12}$	$c_{i,12}$	0	0	X	0	0	0	0	0	0	X
13	$x_{i,13}$	$c_{i,13}$	0	0	0	X	0	0	0	0	0	0
14	$x_{i,14}$	$c_{i,14}$	0	0	0	X	0	0	0	0	0	X
15	$x_{i,15}$	$c_{i,15}$	0	0	0	0	X	0	0	0	0	0
16	$x_{i,16}$	$c_{i,16}$	0	0	0	0	0	X	0	0	0	0
17	$x_{i,17}$	$c_{i,17}$	0	0	0	0	0	0	X	0	0	0
18	$x_{i,18}$	$c_{i,18}$	0	0	0	0	0	0	0	X	0	0
19	$x_{i,19}$	$c_{i,19}$	0	0	0	0	0	0	0	0	X	0
20	$x_{i,20}$	$c_{i,20}$	0	0	0	0	0	0	0	0	0	X

Treatment

Note: X denotes harvesting while 0 denotes no harvesting

Activity Adjacency Matrix

$$\boldsymbol{A}_1^V : (n \times n) \text{ where } a_{i,j}^v = \begin{cases} 1 & \text{if the } i\text{-th and } j\text{-th treatments are concurrent over 2 periods} \\ 0 & \text{otherwise} \end{cases}$$

2 Green-up

$$\boldsymbol{A}_2^V = \begin{pmatrix}
1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&1&1&1&0&0\\
1&1&1&1&1&1&1&1&1&0&1&0&0&0&0&0&1&1&1&0\\
1&1&1&1&1&1&1&1&1&0&1&1&0&1&0&0&0&1&1&1\\
1&1&1&1&1&1&1&1&1&0&1&1&0&1&0&0&0&0&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&1&1&1&0\\
1&1&1&1&1&1&1&1&1&1&1&1&0&1&0&0&0&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&0&1&0&0&0&0&1&1\\
0&0&0&0&0&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0\\
0&0&1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&1&1&1\\
0&0&0&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0\\
0&0&0&1&1&0&0&1&1&1&1&1&1&1&1&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&1&1&0&0&0&1&0&0&0&0&0&0&0&0&1&1&1&0&0\\
0&1&1&1&0&0&1&1&0&0&1&0&0&0&0&0&1&1&1&0\\
0&0&1&1&1&0&1&1&1&0&1&1&0&1&0&0&0&1&1&1\\
0&0&0&1&1&0&0&1&1&0&1&1&0&1&0&0&0&0&1&1
\end{pmatrix}$$

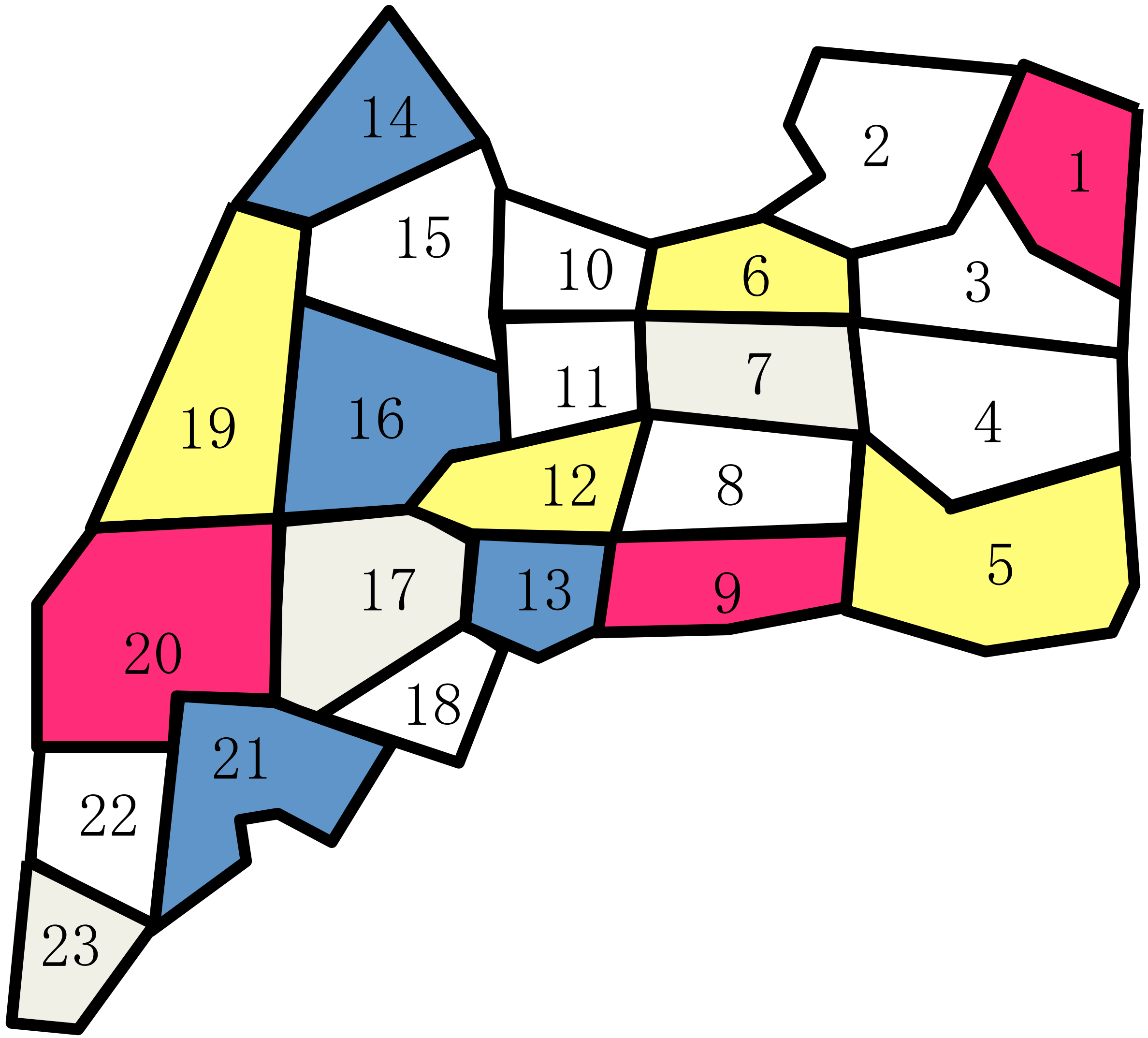
Activity Adjacency Matrix

$$\boldsymbol{A}_3^V : (n \times n) \text{ where } a_{i,j}^v = \begin{cases} 1 & \text{if the } i\text{-th and } j\text{-th treatments are concurrent over 3 periods} \\ 0 & \text{otherwise} \end{cases}$$

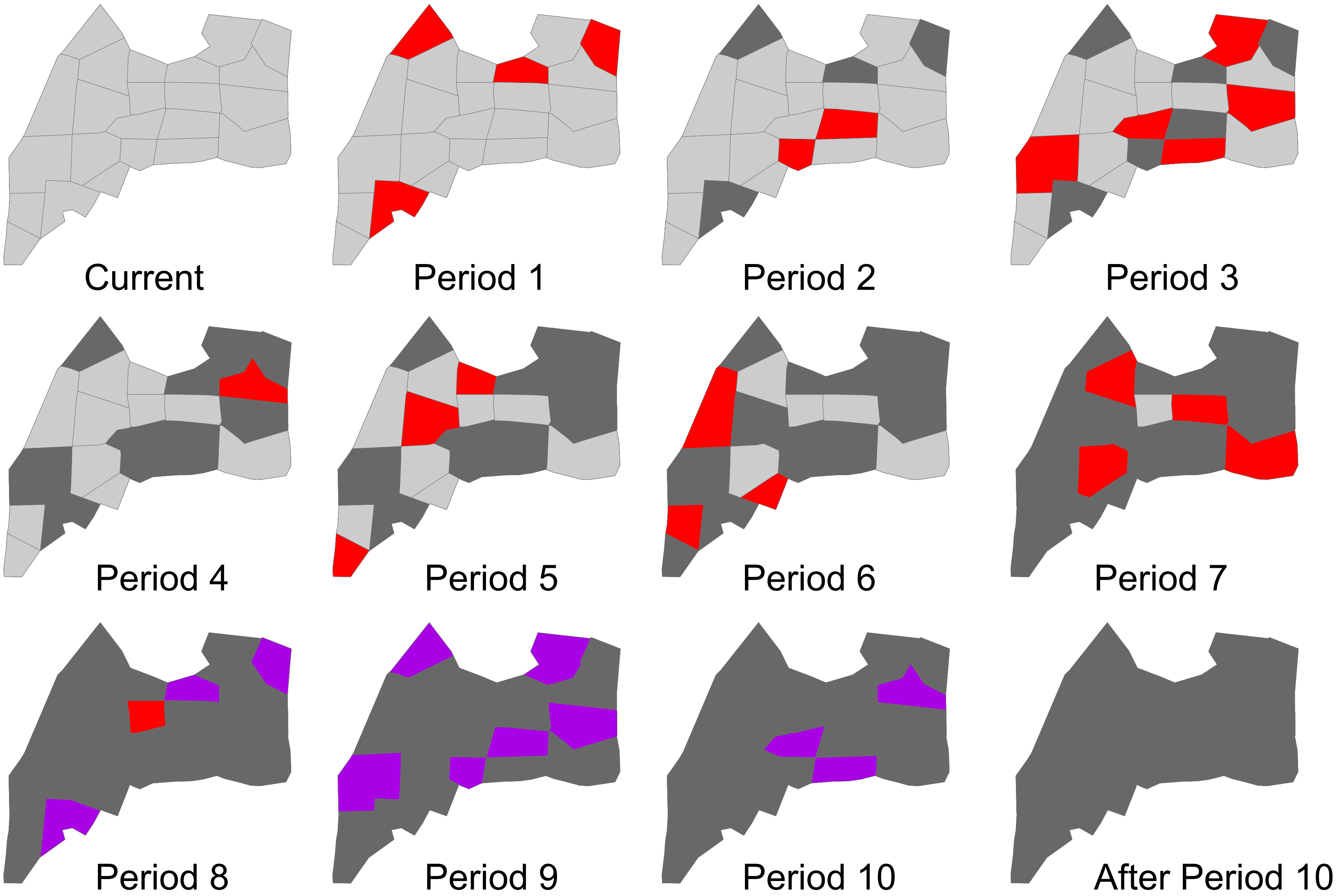
$$\boldsymbol{A}_3^V = \begin{pmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&1&1&1&0&0&1&1&1&1&1&0\\
1&1&1&1&1&1&1&1&1&1&1&1&0&1&0&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&0&1&0&0&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&0&1&0&0&0&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&1&1&1\\
0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0\\
0&0&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&1&1&1\\
0&1&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1&0&0&0\\
0&1&1&0&0&0&1&0&0&0&0&0&1&1&1&1&1&1&0&0\\
0&1&1&1&0&0&1&1&0&0&1&0&0&0&1&1&1&1&1&0\\
0&1&1&1&1&0&1&1&1&0&1&1&0&1&0&1&1&1&1&1\\
0&1&1&1&1&0&1&1&1&0&1&1&0&1&0&0&1&1&1&1\\
0&0&1&1&1&0&1&1&1&0&1&1&0&1&0&0&0&1&1&1
\end{pmatrix}$$

3 Green-up

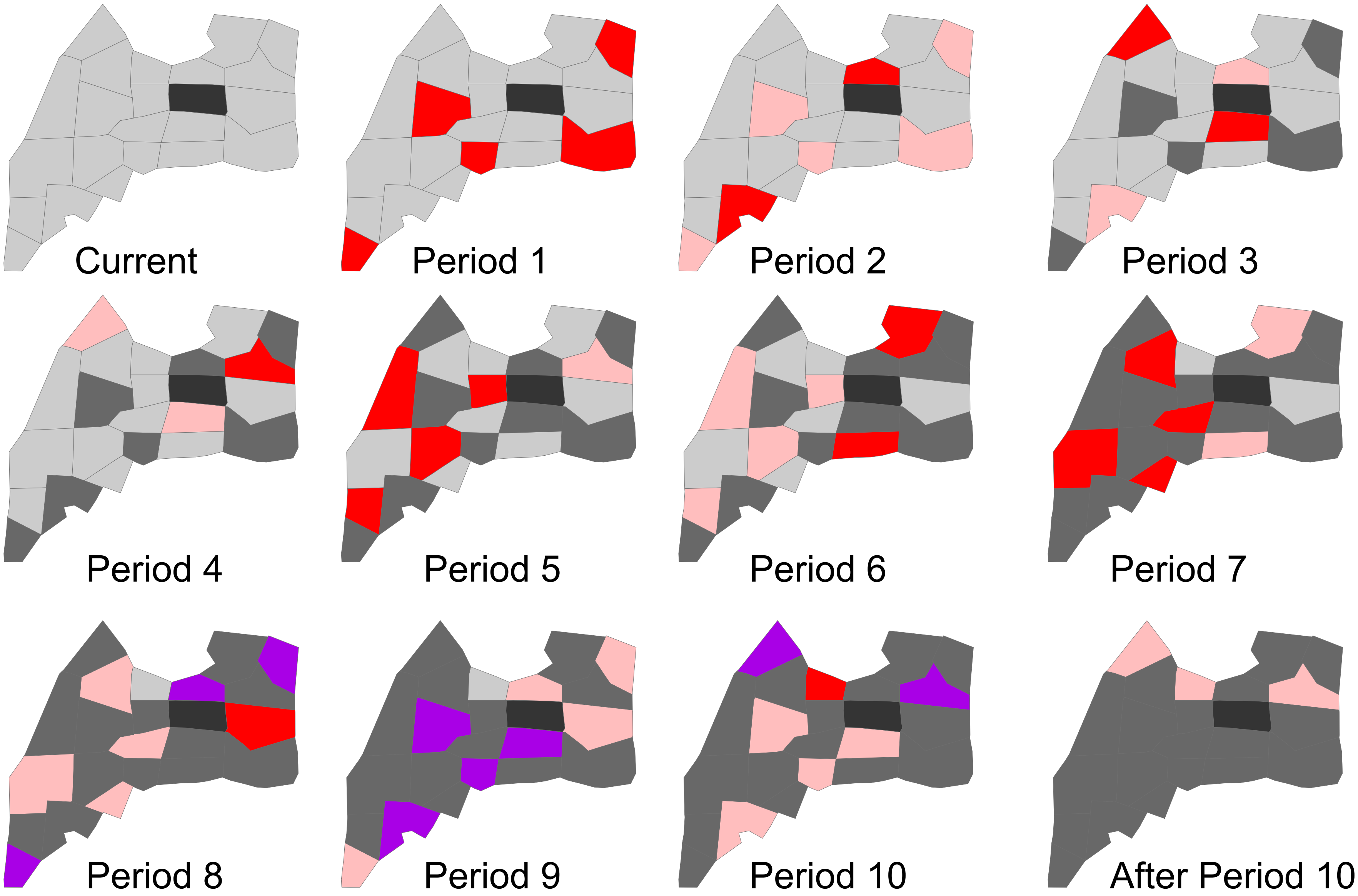
23林分の場合



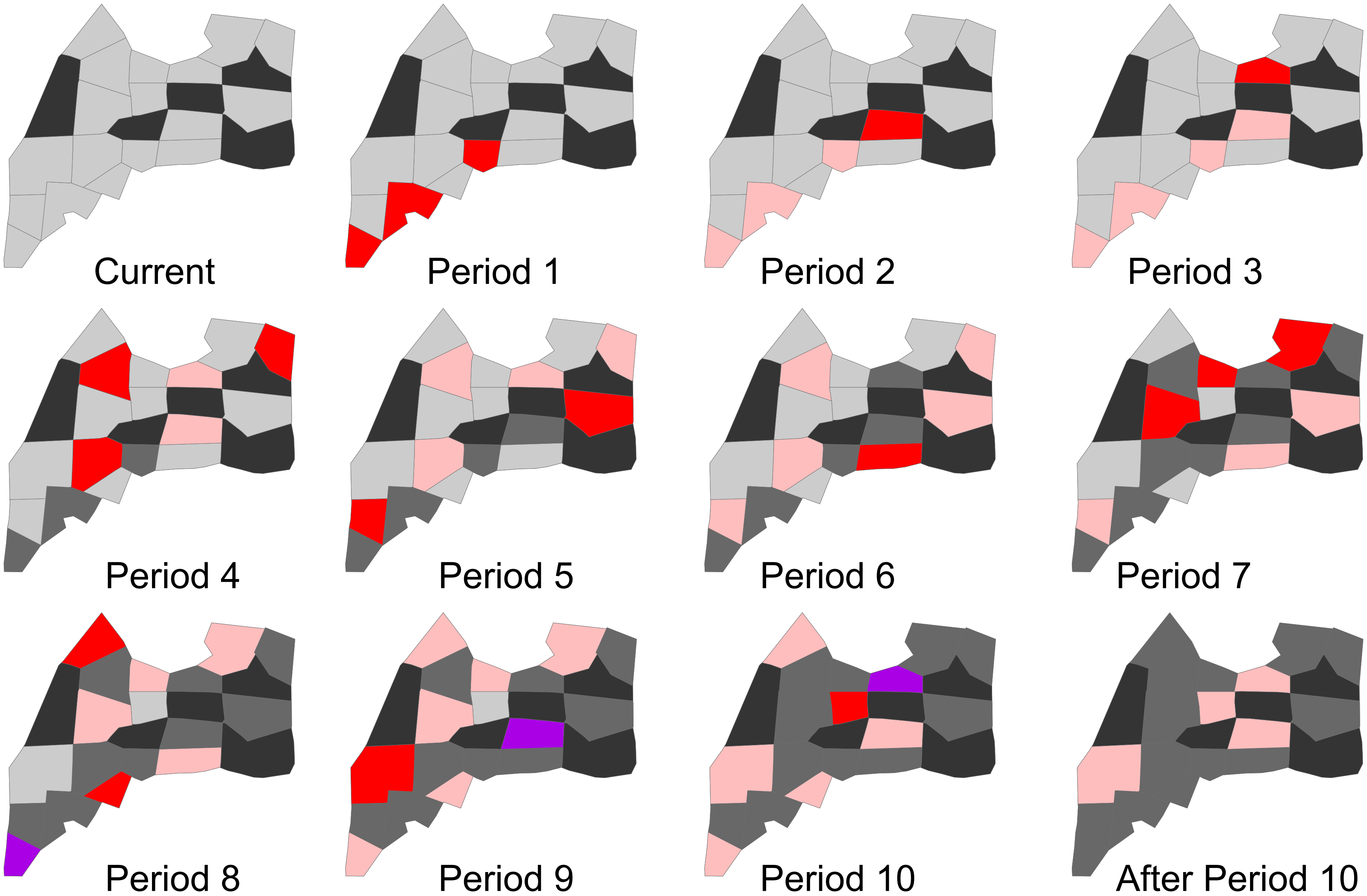
PER10, GUP1

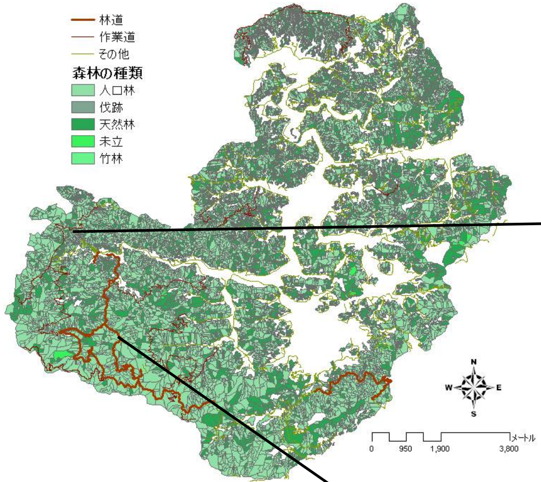

Current
Period 1
Period 2
Period 3
Period 4
Period 5
Period 6
Period 7
Period 8
Period 9
Period 10
After Period 10

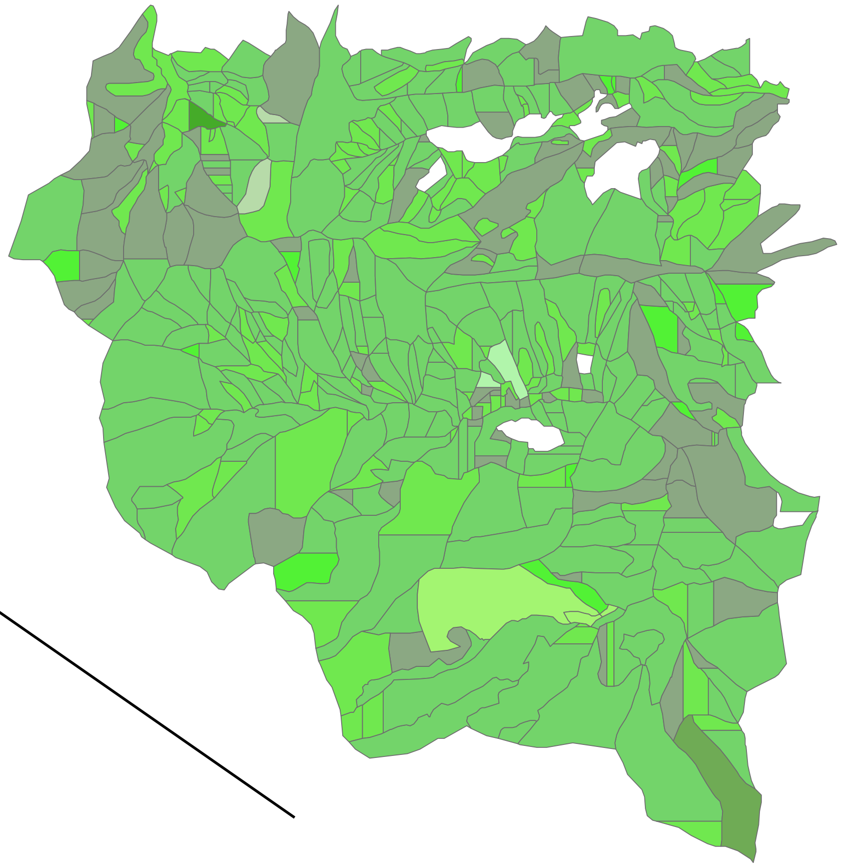
PER10, GUP2


Current
Period 1
Period 2
Period 3
Period 4
Period 5
Period 6
Period 7
Period 8
Period 9
Period 10
After Period 10

PER10, GUP3

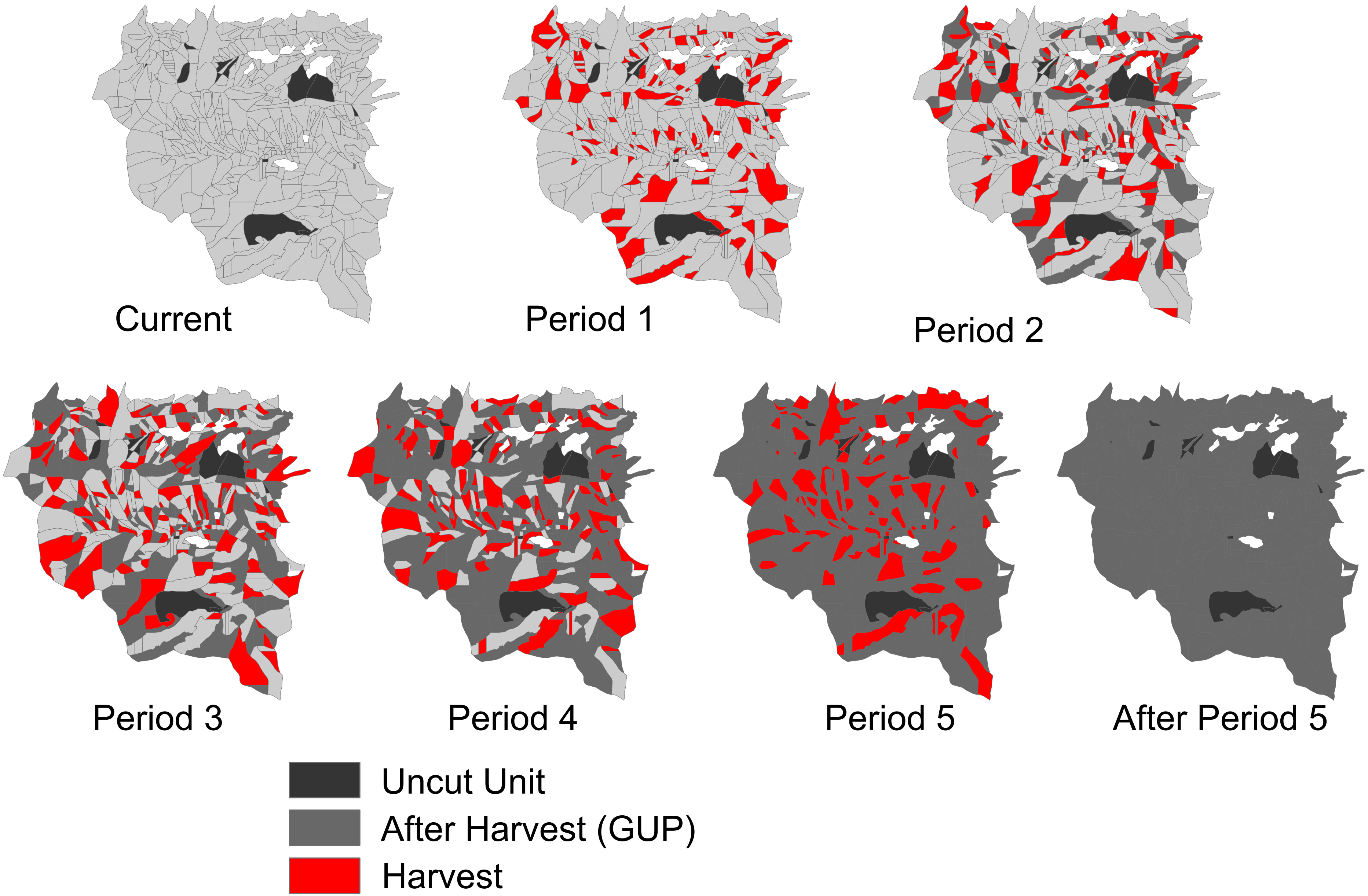

Current
Period 1
Period 2
Period 3
Period 4
Period 5
Period 6
Period 7
Period 8
Period 9
Period 10
After Period 10



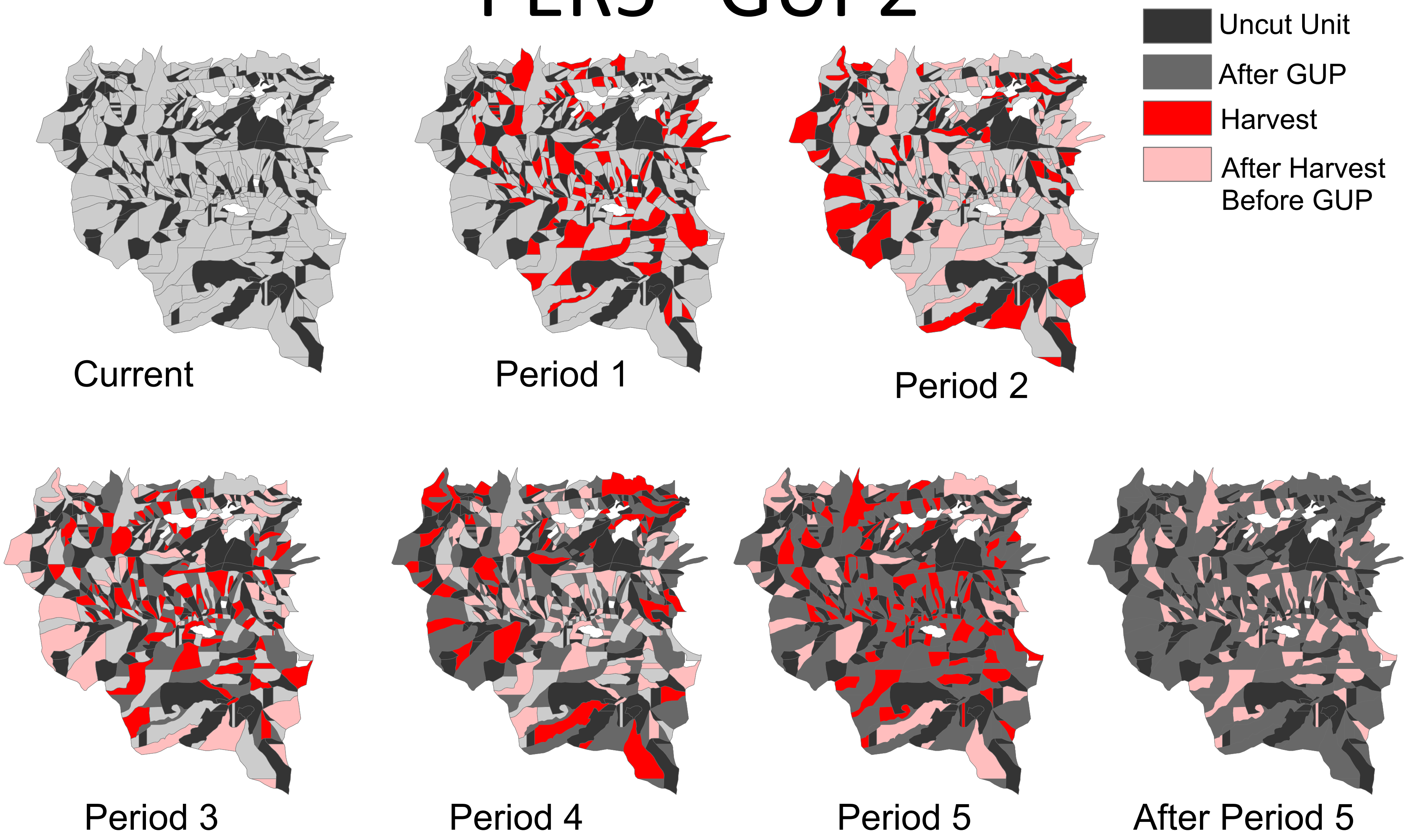


- 462 MUs
- Max. Vol.

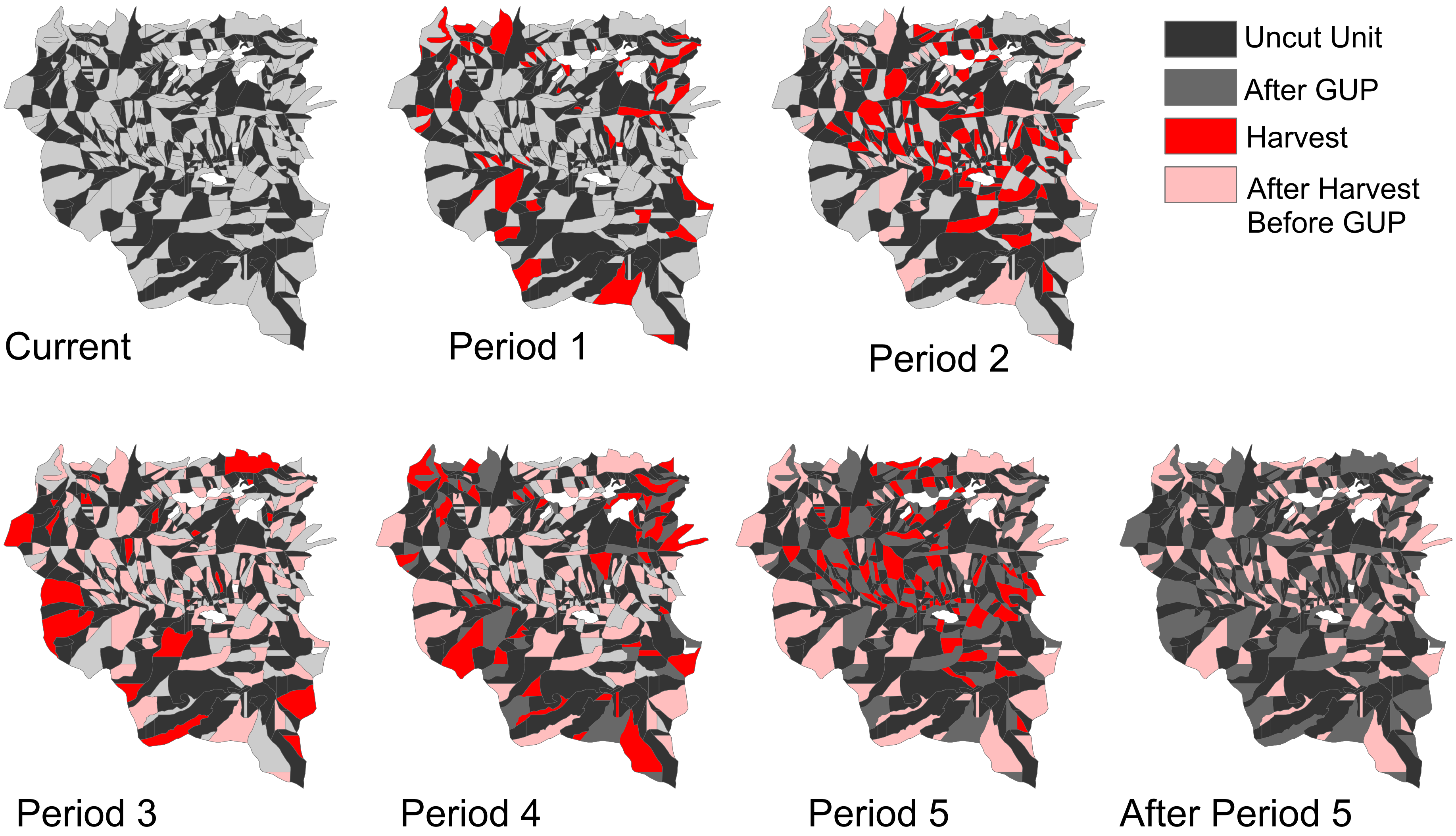
PER5 GUP1


Current
Period 1
Period 2
Period 3
Period 4
Period 5
After Period 5
Uncut Unit
After Harvest (GUP)
Harvest

PER5 GUP2

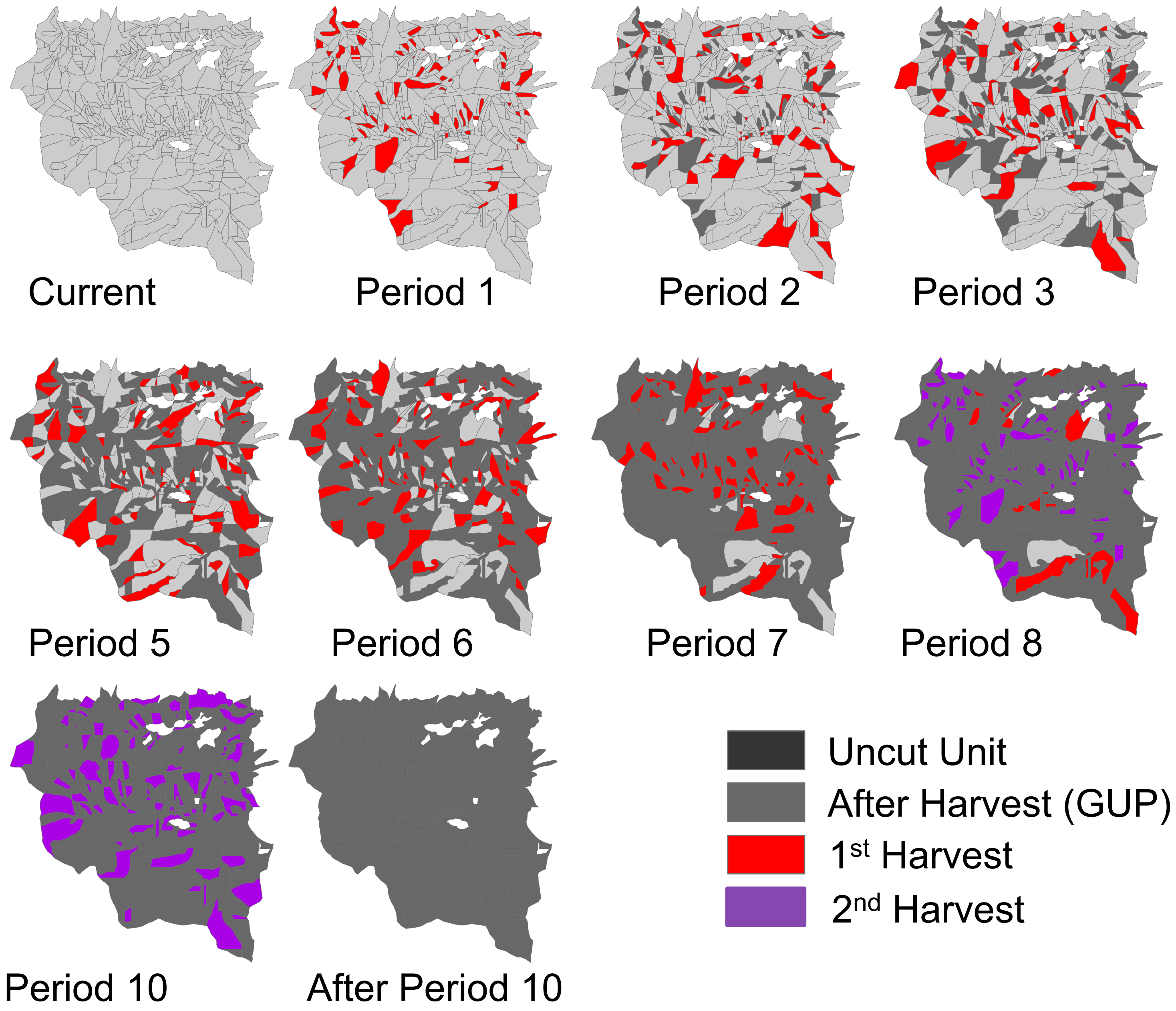

Uncut Unit
After GUP
Harvest
After Harvest Before GUP
Current
Period 1
Period 2
Period 3
Period 4
Period 5
After Period 5

PER5 GUP3

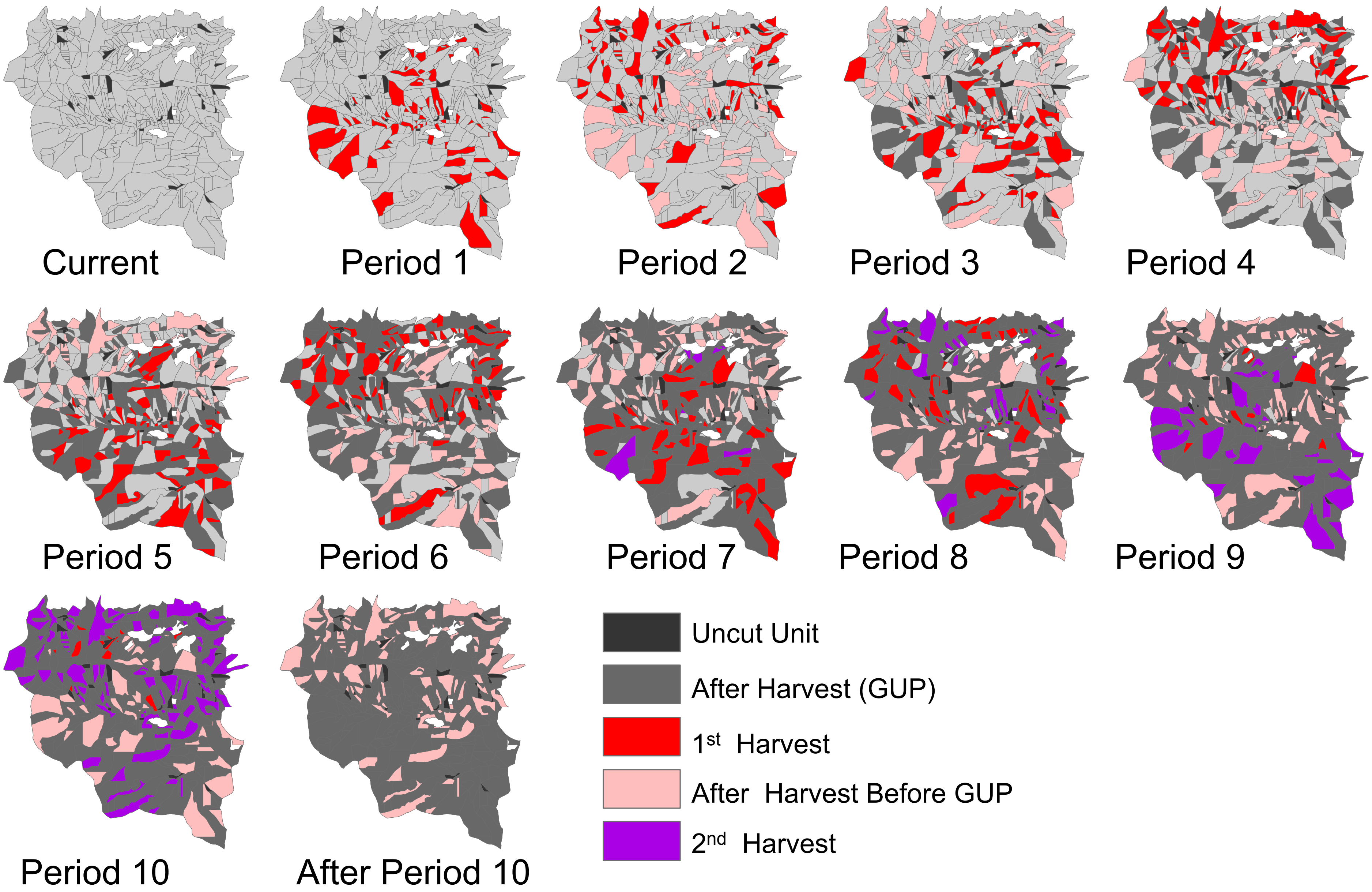


Uncut Unit
After GUP
Harvest
After Harvest Before GUP
Current
Period 1
Period 2
Period 3
Period 4
Period 5
After Period 5

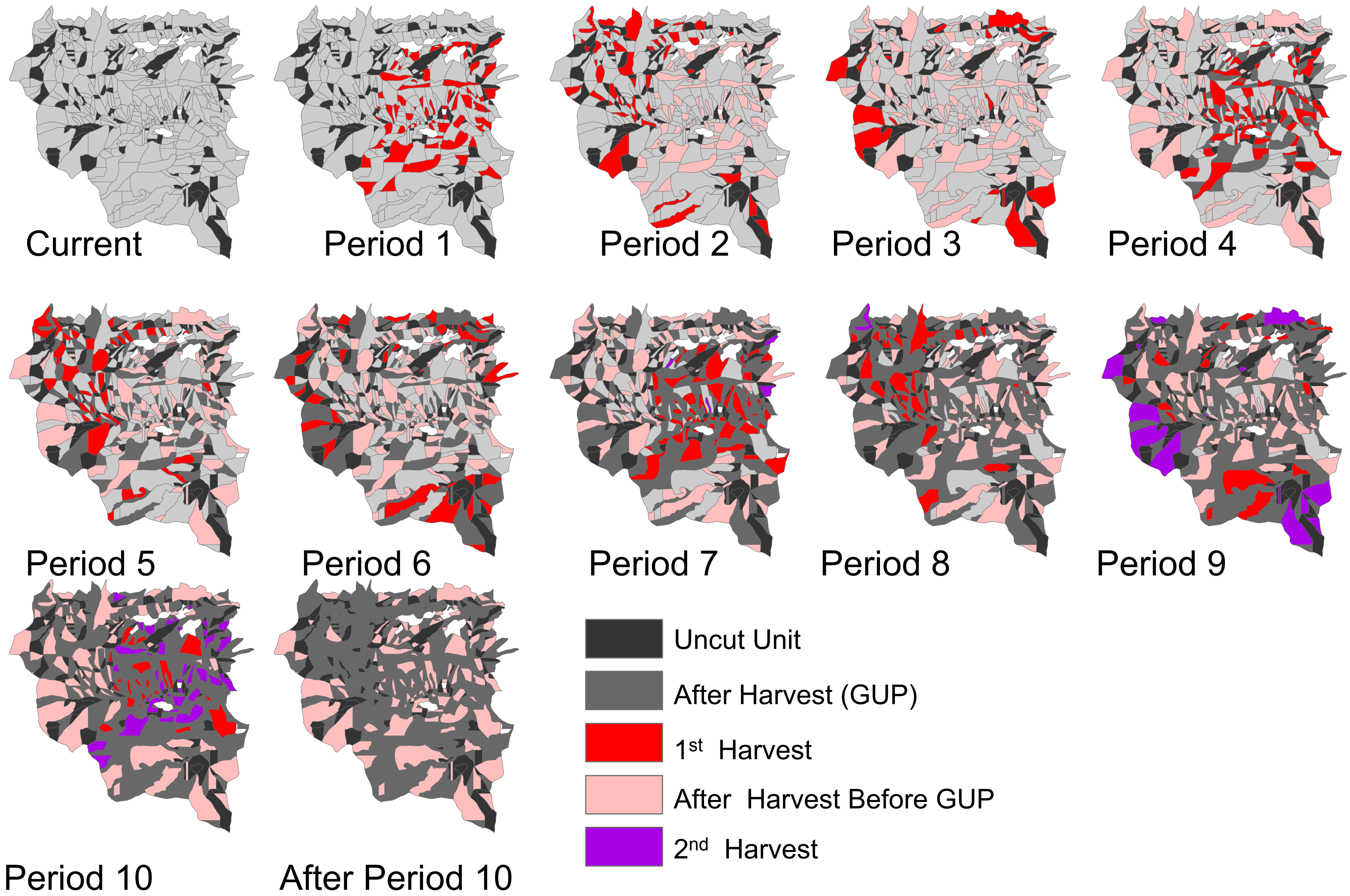
PER10 GUP1


Current
Period 1
Period 2
Period 3
Period 4
Period 5
Period 6
Period 7
Period 8
Period 9
Period 10
After Period 10
Uncut Unit
After Harvest (GUP)
1st Harvest
2nd Harvest

PER10 GUP2


Current
Period 1
Period 2
Period 3
Period 4
Period 5
Period 6
Period 7
Period 8
Period 9
Period 10
After Period 10
Uncut Unit
After Harvest (GUP)
1st Harvest
After Harvest Before GUP
2nd Harvest

PER10 GUP3


Current
Period 1
Period 2
Period 3
Period 4
Period 5
Period 6
Period 7
Period 8
Period 9
Period 10
After Period 10
Uncut Unit
After Harvest (GUP)
1st Harvest
After Harvest Before GUP
2nd Harvest